Practice 2 for Midterm 1

Math 22A, Fall 2019

Name: Solution

Student ID: ____________________

You do not need to simplify numerical expressions for your final answers (e.g. you can write $3 - \frac{3}{4}$ instead of simplifying to $\frac{9}{4}$.)

Problem 1: Calculate the vector given by the following linear combination

$$c \begin{bmatrix} 1 \\ 3 \end{bmatrix} + d \begin{bmatrix} -2 \\ -6 \end{bmatrix}$$

Answer: $\underline{\begin{bmatrix} c - 2d \\ 3c - 6d \end{bmatrix}}$

All linear combinations of

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ -6 \end{bmatrix} \text{ fill:}$$

$x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -6 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} - 2x_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = (x_1 - 2x_2) \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, which is a line.

A. A Line B. A Point C. Two-dimensional space **Answer:** ___A___

All linear combinations of

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 0 \end{bmatrix} \text{ fill:}$$

These two vectors are linearly independent, hence form a plane.

A. A Line B. A Point C. Two-dimensional space **Answer:** ___C___

Problem 2: Calculate the dot product of the following two vectors:

$$\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = 2 \times 1 + 2 \times (-2) + (-1) \times (-1) = -1$$

Answer: ____-1____

Fill in the blank:
If u and v are vectors and their dot product $u \cdot v = 0$, then u and v are ________.

Answer: ____orthogonal/perpendicular____

Is the angle between these two vectors

$$\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

The dot product of these two vectors is negative, -1 from previous calculation, hence the angle between them is greater than 90° and less than 180°.

A. Less than 90° C. Greater than 90° and less than 180°
B. Equal to 90° D. Equal to 180°.

Answer: ____C____

Calculate the length of the vector

$$\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$$\text{length} = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14}$$

Length: ____$\sqrt{14}$____

Which matrix multiplications are possible? (choose all that apply)

A. $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$ **B.** $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

columns of the left matrix= # rows of the right matrix

Answer: ____A C____

Problem 3: Write out the following system of equations in matrix form $Ax = b$:

$$\begin{aligned} 2x_1 - x_2 \qquad &= 1 \\ + x_2 + 3x_3 &= 2 \\ 4x_1 - x_2 + 3x_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 3 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Which matrix E will do: replace the 3^{rd} row by the (3^{rd} row) – 2(1^{st} row) when multiplying EA?

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

What is the result of multiplying both sides of the equation above by E ($EAx = Eb$)? (Do what E does to both sides.)

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

Perform another row operation on both sides to change the resulting matrix to have all 0's below the diagonal. What is the result?

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$$

Which matrix F did you use to do that row operation?

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Does the system of equations have

A. One solution
B. Infinitely many solutions
C. No solutions

Answer: C

Problem 4: State whether or not the following matrices have an inverse or not:

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

This matrix is not square and therefore cannot have an inverse.

A. Has an inverse **B.** No inverse **Answer:** B

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

This matrix is in upper triangular form and has a 0 on the diagonal. Therefore it has no inverse.

A. Has an inverse **B.** No inverse **Answer:** B

Calculate A^{-1} if A is the matrix

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 3 & 2 & 0 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & 0 & \frac{1}{3} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & 0 & \frac{1}{3} \end{bmatrix}$$

Answer: $\begin{bmatrix} -\frac{1}{3} & 0 & \frac{1}{3} \\ 0 & -1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \end{bmatrix}$

Problem 5: Solve for the vector x:
Solve $Ax = b$ if

$$A = \begin{bmatrix} -17 & -7 & 4 \\ 5 & 2 & -1 \\ -3 & -1 & 1 \end{bmatrix}, \ b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \ \text{ and } A^{-1} = \begin{bmatrix} 1 & 3 & -1 \\ -2 & -5 & 3 \\ 1 & 4 & 1 \end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} 1 & 3 & -1 \\ -2 & -5 & 3 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 6 \end{bmatrix}$$

Answer: $\begin{bmatrix} 3 \\ -4 \\ 6 \end{bmatrix}$

Find the line of solutions solving $Ax = b$ if

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}, \ \text{ and } b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

Let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, doing row elimination for $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 2 & 2 & 2 & 4 \end{bmatrix}$ we get $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
Expressing in terms of x_1 x_2 x_3 we get

$$\begin{cases} x_1 + x_2 + x_3 = 2 \\ \quad\quad x_2 + 2x_3 = 3 \end{cases} \implies \begin{cases} x_1 = -1 + x_3 \\ x_2 = 3 - 2x_3 \end{cases}$$

Thus,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 = -1 + x_3 \\ x_2 = 3 - 2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Since x_3 is a free variable, by setting $x_3 = t + 1$, we get

$$x = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} + (t+1) \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Answer: $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$