Practice Midterm 2

Math 22A, Fall 2019

Name: ______________________________________

Student ID: ________________________________

You do not need to simplify numerical expressions for your final answers (e.g. you can write $3 - \frac{3}{4}$ instead of simplifying to $\frac{9}{4}$.)

$$\frac{\mathbf{v} \cdot \mathbf{w}}{||\mathbf{v}||\,||\mathbf{w}||} = \cos\theta$$

The formula for the projection matrix is

$$P = A(A^T A)^{-1} A^T$$

Problem 1 (pts): Determine whether each of the following sets is a subspace of P_2 the polynomials of degree 2. If it is a subspace, prove it is closed under scalar multiplication and addition. If it is not a subspace give an example showing it is not closed under one of the two operations.

(a) The set of polynomials of the form $p(t) = at^2$ where a is in $\mathbb{R}$.

(b) The set of polynomials of the form $p(t) = t^2 + a$ where a is in $\mathbb{R}$.

Problem 2 (pts):

(a) Find the value of k for which the matrix

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & -1 & 2 \\ 1 & 1 & k \end{bmatrix}$$

has rank 2.

(b) Give an example of a matrix whose column space contains $(1, 2, 5)$ and $(0, 4, 1)$ and whose null space contains $(1, -1, 2)$.

Problem 3 (pts): State whether the following vectors are linearly independent or dependent. If they are linearly independent, prove it. If they are linearly dependent, give coefficients c_1, c_2, c_3, c_4 which are not all zero such that
$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$.

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Problem 4 (pts): Find bases for the null space $N(A)$ and the column space $C(A)$.

$$A = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 1 & 3 & 0 & 3 \end{bmatrix}$$

Problem 5 (pts):

1. Amongst the following subspaces, specify all pairs which are orthogonal to each other.

$$U_1 = \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right), \quad U_2 = \text{Span}\left(\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}\right), \quad U_3 = \text{Span}\left(\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}\right),$$

$$U_4 = \text{Span}\left(\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}\right), \quad U_5 = \text{Span}\left(\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}\right), \quad U_6 = \text{Span}\left(\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}\right)$$

2. Calculate the projection matrix which projects vectors onto the following subspace

$$U = \text{Span}\left(\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \end{bmatrix}, \begin{bmatrix} \frac{-1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \\ 0 \end{bmatrix}\right)$$

Problem 6 (pts): Let

$$A = \begin{bmatrix} 3 & 1 & 3 \\ 0 & 2 & 6 \\ 4 & -1 & -3 \end{bmatrix}$$

Find an *orthonormal basis* for the column space of A.

Problem 7 (pts):

1. Answer whether each of the following statements is true or false:

   (a) The determinant of $I + A$ is $1 + \det(A)$. **(True) / (False)**

   (b) The determinant of ABC is $\det(A)\det(B)\det(C)$. **(True) / (False)**

   (c) The determinant of $4A$ is $4\det(A)$. **(True) / (False)**

2. Determine the value of k which ensures the following matrix has $\det(A) = 5$:

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 \\ 1 & 1 & 2 & k \end{bmatrix}$$