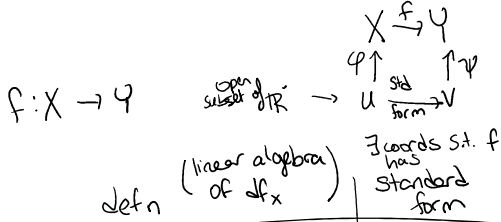
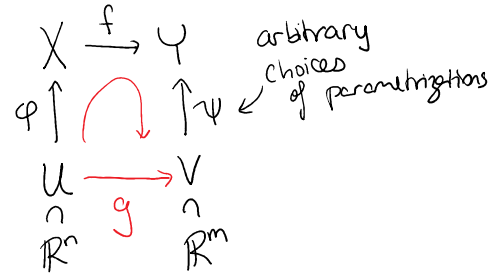


$$f: X \rightarrow Y$$

X, Y smooth manifolds

f smooth $\xrightarrow{\text{defn}}$

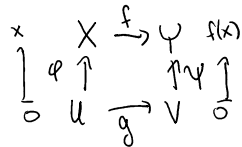


Say f is smooth if g is smooth in the usual analysis sense

	defn (linear algebra of df_x)	\exists coords s.t. f has standard form	Preimage theorems
immersion	df_x injective $\forall x \in X$	$i: \mathbb{R}^k \rightarrow \mathbb{R}^n$ $k \leq n$ $(x_1, \dots, x_k) \mapsto (x_1, \dots, x_k, 0, \dots, 0)$	
submersion	df_x surjective $\forall x \in X$	$p: \mathbb{R}^m \rightarrow \mathbb{R}^n$ $m \geq n$ $(x_1, \dots, x_m) \mapsto (x_1, \dots, x_n)$	If y is a regular value $f^{-1}(y)$ is a submanifold of X $\dim(f^{-1}(y)) = \dim X - \dim Y$
local diffeo	df_x isomorphism $\forall x \in X$	identity	
transverse to $Z \subset Y$ \uparrow submanifold	$\text{image}(df_x) + T_{f(x)}Z = T_{f(x)}Y$ $\forall x \in f^{-1}(Z)$	$Z = (x_1, \dots, x_k, 0, \dots, 0)$ $i: \mathbb{R}^m \rightarrow \mathbb{R}^n$ $(x_1, \dots, x_m) \mapsto (0, \dots, 0, x_1, \dots, x_k)$ $n-k \leq k$	$f \nmid Z$ $f^{-1}(Z)$ is a submfd of X $\dim(f^{-1}(Z)) = \dim X - \dim Y + \dim Z$

$$g := \psi^{-1} \circ f \circ \phi$$

To prove local forms:

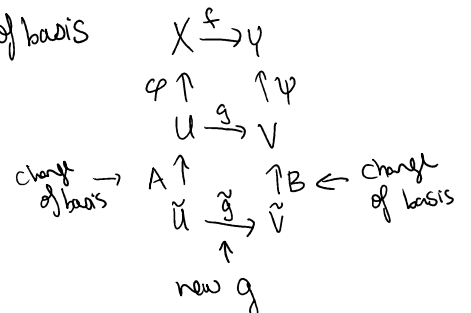


$g = \psi^{-1} \circ f \circ \phi$ Goal to change ϕ, ψ until g is standard model

- Look at dg_0 , The defining property + linear algebra $\Rightarrow \exists$ change of basis to transform dg_0 into a matrix that looks like $d(\text{standard model})$.
 \uparrow
Euclidean (matrix) version of df_x

Use change of basis transformations, thinking of them as smooth maps $\mathbb{R}^i \rightarrow \mathbb{R}^i$, to modify ϕ, ψ by composing w/ change of basis

Now with new $g(\tilde{g})$, $dg_0 = d(\text{standard model})$



Want to use inverse function theorem

Found G defined between open subsets of \mathbb{R}^i such that

- G composes (in some order) with std model to equal g .
- dG_0 is an isomorphism

Then use inverse fct thm to say G is a local diffeo,

compose either φ or ψ with G to get better φ or ψ

- because G local diffeo, still get parametrization after composing φ/ψ with G/G^{-1}
- because G composed w/ std model to give g

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \varphi \uparrow & & \uparrow \psi \\ U & \xrightarrow{g} & V \end{array}$$

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \varphi \uparrow & \text{depending} & \uparrow \psi \\ U & \xrightarrow{G} & W \xrightarrow{\text{std}} V \end{array}$$

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \uparrow \varphi & & \uparrow \psi \\ W & \xrightarrow{\text{std}} & V \end{array}$$

Preimage theorems:

Regular value case:

$f^{-1}(y)$

Standard model for submersion, preimage of a point

$$m \geq n \quad p: (x_1, \dots, x_m) \mapsto (x_1, \dots, x_n)$$

$$\begin{aligned} p^{-1}(0, \dots, 0) &= \{(0, \dots, 0, x_{n+1}, \dots, x_m) \mid x_{n+1}, \dots, x_m \in \mathbb{R}\} \\ &= 0 \times \mathbb{R}^{m-n} \\ &\cong \mathbb{R}^{m-n} \\ &\mathbb{R}^{\dim X - \dim Y} \end{aligned}$$

Transverse case: Reduced to regular value case

Locally represent $Z \subset Y$ as $(x_1, \dots, x_k, 0, \dots, 0)$

$$\begin{array}{ccc} Y & & \\ \downarrow & & \\ W & & \\ \downarrow & & \\ \mathbb{R}^n & & \end{array}$$

$$\psi^{-1}: \overset{\varphi}{W} \rightarrow \overset{\mathbb{R}^n}{V}$$

$$\psi^{-1}(y) = (g_1(y), \dots, g_n(y))$$

$$Z \cap W = \{g_{k+1}(y) = \dots = g_n(y) = 0 \mid y \in W \subset Y\}$$

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ U & & U \\ f^{-1}(W) & \xrightarrow{f} & W \xrightarrow{g} \mathbb{R}^{n-k} \end{array}$$

$$g(y) = (g_{k+1}(y), \dots, g_n(y))$$

Transverse condition for f , + fact that df_x is surjective \Rightarrow $g \circ f$ submersion
at $x \in f^{-1}(z) \cap W$

Use regular value preimage theorem.