MAT 239: DIFFERENTIAL TOPOLOGY

This course is meant to introduce the fundamental tools of to study smooth manifolds.
Textbook: Differential Topology by Guillemin and Pollack

1. Assignments

There will be weekly homework assignments due on **Fridays**. Please submit your completed assignments through Gradescope.
You are encouraged to discuss the problems with your peers, but you must gain your own understanding and write up your solutions in your own words. You may use the course textbooks as references. You should provide complete proofs, utilizing the results we prove in class.
If you use any outside sources other than the course textbooks, they must be cited. Use of any kind of so called "tutoring" websites like Chegg is not permitted. You are forbidden from posting my problems to any website and use of Chegg or any similar site will be considered cheating and a violation of the honor code. Cheating in a graduate class does not make any sense, but if it occurs, there is a no tolerance policy and it will be reported to OSSJA.
As this course is intended to provide preparation for the prelim exams, there will be one timed midterm and one timed final exam.
Your course grade will be determined by these assignments with the following weights:

- 60% Homework
- 15% Midterm
- 25% Final

2. Topics

We will cover the topics included in the prelim syllabus which are as follows. (The course may cover them in a slightly different order.)

(1) Fundamental definition and examples
  - Smooth manifold, charts, transition functions
  - Dimension
  - Verifying examples are smooth manifolds (e.g. the sphere, products of manifolds)
  - Smooth maps between smooth manifolds
  - Smooth manifold with boundary

(2) The tangent bundle and the derivative of a smooth map between manifolds
  - Tangent space, definition and finding in examples
  - Jacobian, Chain rule
  - Rank of determinant, Immersion, Submersion
  - Inverse function theorem
  - Local model for immersion
  - Implicit function theorem
  - Local model for submersion

(3) Regular values
  - Regular values (definition and examples)
  - Preimage of a regular value
  - Examples of manifolds cut out by equations
  - Tangent space to manifolds cut out by equations
  - Sard's theorem (statement and applications, not proof)

(4) Morse Theory
  - Morse's Lemma

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- Genericity of Morse functions

(5) Transversality
- Definition of a map or submanifold being transverse to a submanifold
- Preimage of a submanifold under a transverse map
- Dimension formula for transverse intersections
- Normal bundle
- Stability of transverse maps
- Genericity of transverse maps

(6) Orientation
- Orientation and orientability
- Boundary orientation
- Preimage orientation

(7) Intersection number
- Mod 2 intersection number
- Oriented intersection number
- Euler characteristic as oriented intersection number
- Poincare-Hopf theorem

(8) Vector fields
- Definition and examples
- Index of the zeros of a vector field
- Euler characteristic in terms of zeros of a vector field

(9) Differential forms
- Definitions, computations, examples of k-form, wedge product, exterior derivative, pull-back
- Integrating k-forms, change of variables
- Properties of exterior derivative (e.g. Leibniz rule, commutes with pull-backs)
- deRham cohomology
- Stokes Theorem

3. Accommodations

UC Davis is committed to educational equity in the academic setting, and in serving a diverse student body. I encourage all students who are interested in learning more about the Student Disability Center (SDC) to contact them directly at sdc.ucdavis.edu, sdc@ucdavis.edu or 530-752-3184. You are also encouraged to talk to me directly about anything that would help facilitate your learning.