SHORT CALCULUS Math 16C Sec 2 Spring 2008
Homework #5 Solutions
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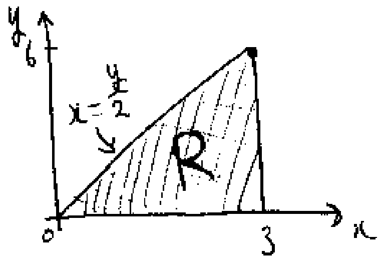
Section 7.9

Question 4

The region of integration is $R = \{(x,y) : 0 \le y \le 6, \frac{y}{2} \le x \le 3\}$.

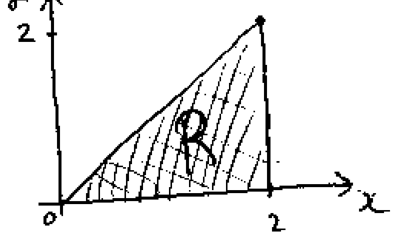


$$\int_0^6 \int_{\frac{y}{2}}^3 (x+y)dxdy = \int_0^6 \left[\frac{1}{2}x^2 + xy\right]_{\frac{y}{2}}^3 = \int_0^6 \left(\frac{1}{2}3^2 + 3y - \frac{1}{2}\left(\frac{y}{2}\right)^2 - \frac{y}{2}\cdot y\right) dy$$
$$= \int_0^6 \left(\frac{9}{2} + 3y - \frac{5}{8}y^2\right) dy = \left[\frac{9}{2}y + \frac{3}{2}y^2 - \frac{5}{24}y^3\right]_0^6$$
$$= (27 + 54 - 45) - (0) = 36$$

Question 18

The region of integration is

$$R = \{(x,y) : 0 \le x \le 2, 0 \le y \le x\} = \{(x,y) : 0 \le y \le 2, y \le x \le 2\}.$$



The volume of the specified solid is

$$V = \int_0^2 \int_0^x 6dydx = \int_0^2 [6y]_0^x = \int_0^2 6xdx = \left[3x^2\right]_0^2 = 12$$

or

$$V = \int_0^2 \int_y^2 6dxdy = \int_0^2 [6x]_y^2 \, dy = \int_0^2 (12-6y)dy = \left[12y - 3y^2\right]_0^2$$
$$= (24-12) - (0) = 12.$$

Question 24

The region of integration is

$$R = \{(x,y) : 0 \le x, 0 \le y\}.$$

The diagram in the book is a little misleading since the x and y variables are not both bounded above by 2. Note that we need to use improper integrals to solve this problem.

$$\begin{aligned}
V &= \int_0^\infty \int_0^\infty \frac{1}{(x+1)^2(y+1)^2} dxdy \\
&= \int_0^\infty \left(\lim_{b\to\infty} \int_0^b \frac{1}{(x+1)^2(y+1)^2} dx \right) dy \\
&= \int_0^\infty \left(\lim_{b\to\infty} \left[\frac{-1}{(x+1)(y+1)^2} \right]_0^b \right) dy \\
&= \int_0^\infty \left(\lim_{b\to\infty} \frac{-1}{(b+1)(y+1)^2} + \frac{1}{(y+1)^2} \right) dy \\
&= \int_0^\infty \frac{1}{(y+1)^2} dy \\
&= \lim_{b\to\infty} \int_0^b \frac{1}{(y+1)^2} dy \\
&= \lim_{b\to\infty} \left[\frac{-1}{(y+1)} \right]_0^b \\
&= \lim_{b\to\infty} \frac{-1}{(b+1)} + 1 \\
&= 1
\end{aligned}$$

If we reverse the order of integration, then the working out is the same after swapping x and y.

Question 26

$$\begin{aligned}
V &= \int_0^\infty \int_0^\infty e^{-(x+y)/2} dydx \\
&= \int_0^\infty \lim_{b\to\infty} \int_0^b e^{-(x+y)/2} dydx \\
&= \int_0^\infty \lim_{b\to\infty} \left[-2e^{-(x+y)/2} \right]_0^b dx \\
&= \int_0^\infty 2e^{-x/2} dx \\
&= \lim_{b\to\infty} \int_0^b 2e^{-x/2} dx \\
&= \lim_{b\to\infty} \left[-4e^{-x/2} \right]_0^b = 4
\end{aligned}$$

If we reverse the order of integration, then the working out is the same after swapping x and y.

Question 34

We must find the average value of the function $f(x,y) = xy$ over the region $R = \{(x,y) : 0 \le x \le 4, 0 \le y \le 2\}$. The area of R is 8. So, the average value of f over R is as follows:

$$\begin{aligned}\frac{1}{8}\int_0^2\int_0^4 xydxdy &= \frac{1}{8}\int_0^2 \left[\frac{1}{2}x^2y\right]_0^4 dy \\ &= \frac{1}{8}\int_0^2 8ydy \\ &= \frac{1}{8}\left[4y^2\right]_0^2 \\ &= \frac{1}{8}16 = 2.\end{aligned}$$

So, the average value of f over R is 2.

Question 38

We want to find the average value of the function

$$P(x_1, x_2) = 192x_1 + 576x_2 - x_1^2 - 5x_2^2 - 2x_1x_2 - 5000$$

over the region $R = \{(x_1, x_2) : 40 \le x_1 \le 50, 45 \le x_2 \le 50\}$.

Firstly, since R is a rectangular region, its area is $10 \times 5 = 50$.

Then, the average value of P over R is

$$\begin{aligned}&\frac{1}{50}\int_{45}^{50}\int_{40}^{50}(192x_1 + 576x_2 - x_1^2 - 5x_2^2 - 2x_1x_2 - 5000)dx_1dx_2 \\ =&\frac{1}{50}\int_{45}^{50}\left[96x_1^2 + 576x_1x_2 - \frac{1}{3}x_1^3 - 5x_1x_2^2 - x_1^2x_2 - 5000x_1\right]_{40}^{50} dx_2 \\ =&\frac{1}{50}\int_{45}^{50}\left(\frac{48200}{3} + 4860x_2 - 50x_2^2\right)dx_2 \\ =&\frac{1}{50}\left[\frac{48200}{3}x_2 + 2430x_2^2 - \frac{50}{3}x_2^3\right]_{45}^{50} \\ =&13400.\end{aligned}$$

Section 10.1

Question 14

This sequence converges.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} = \lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = \lim_{n\to\infty} \sqrt{\frac{1}{1+\frac{1}{n}}} = \sqrt{\frac{1}{1}} = 1.$$

Question 16

This sequence converges.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+2}{n^2+1} = \lim_{n\to\infty} \frac{\frac{1}{n}+\frac{2}{n^2}}{1+\frac{1}{n^2}} = \frac{0+0}{1+0} = 0.$$

Question 18

Since $a_n = 1 + (-1)^n$, the sequence is $\{a_n\} = \{0, 2, 0, 2, 0, 2, 0, ...\}$. So, the sequence diverges.

Question 20

This sequence converges.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{n!}{(n+1)\cdot n!} = \lim_{n\to\infty} \frac{1}{(n+1)} = 0.$$

Question 32

The sequence is $3, 7, 11, 15, ...$ The nth term is thus $a_n = 4n - 1$.

Question 34

The sequence is $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, ...$ The nth term is thus $a_n = \frac{1}{n^2}$.

Question 36

The sequence is $2, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, ...$ The nth term is thus $a_n = \frac{n+1}{2n-1}$.

Question 38

The sequence is $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, ...$ The nth term is thus $a_n = \frac{2^{n-1}}{3^n}$.

Question 42

The sequence is $2, -4, 6, -8, 10, ...$ The nth term is thus $a_n = (-1)^{n+1}(2n)$.

Question 60

The nth term is $A_n = P(1+r)^n$ where $P = 5000$ and $r = 0.1$.

1. $A_1 = P(1+r)^1 = 5000(1+0.1)^1 = 5500$.
2. $A_2 = P(1+r)^2 = 5000(1+0.1)^2 = 6050$.
3. $A_3 = P(1+r)^3 = 5000(1+0.1)^3 = 6655$.
4. $A_4 = P(1+r)^4 = 5000(1+0.1)^4 = 7320.5$.
5. $A_5 = P(1+r)^5 = 5000(1+0.1)^5 = 8052.55$.
6. $A_6 = P(1+r)^6 = 5000(1+0.1)^6 \approx 8857.81$.
7. $A_7 = P(1+r)^7 = 5000(1+0.1)^7 \approx 9743.59$.
8. $A_8 = P(1+r)^8 = 5000(1+0.1)^8 \approx 10717.94$.
9. $A_9 = P(1+r)^9 = 5000(1+0.1)^9 \approx 11789.74$.
10. $A_{10} = P(1+r)^{10} = 5000(1+0.1)^{10} \approx 12968.71$.