

Math 16C Sec 2 (Malkin)
Mid-term exam 1
Wed April 30th 2008

Name:
Student ID:
Signature:

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UNTIL INSTRUCTED TO DO SO!**

Write your name, student ID, and signature NOW!

**NO NOTES, CALCULATORS, OR BOOKS ARE ALLOWED.
NO ASSISTANCE FROM CLASSMATES IS ALLOWED.**

Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. Be organized and neat, and use notation appropriately. You will be graded on the proper use of derivative and integral notation. Put units on answers where appropriate.

Please write legibly!!

#	Student's Score	Maximum possible Score
1		15
2		13
3		12
4		10
Total points		50

1. (a) (3 points) Verify that the equation $y = e^{x^3}$ is a solution the differential equation $y'' - 3x^2y' - 6xy = 0$.

(b) (6 points) Use any method to find the general solution of the differential equation $y' + 2xy = x^3e^{-x^2}$. You must write y as a function of x .

(c) (6 points) Use any method to find the general solution of the differential equation $\frac{dy}{dx} = 3x^2(y - 1)^2$. You must write y as a function of x .

2. (a) (3 points) Sketch the plane given by the equation $2x + 3y + z = 6$.

(b) (5 points) Consider the function $f(x, y) = 2 + \sqrt{y - x^2}$. Find the *domain* and *range* of f . *Sketch* the domain.

(c) (5 points) Sketch the level curves of the surface $z = y^2 - x^2$ for $z = -1, 0, 1$.

3. (a) (2 points) Find the slope of the surface $z = f(x, y) = \frac{2x^2}{y}$ at the point $(1, 1, 2)$ in the x -direction and in the y -direction.

- (b) (5 points) Find all the critical points of the function $f(x, y) = 2x^3 + y^2 - 6xy$.
(Hint: there are two critical points).

- (c) (5 points) The function $f(x, y) = x^3 - y^3 + 3xy$ has two critical points: $(0, 0)$ and $(1, -1)$. Classify each of the critical points of f .

4. (10 points) A rumour is spreading through the student population of UC Davis. Let S be the percentage of students who have heard the rumour after t days ($0 \leq S \leq 100$). The rate at which the rumour spreads is proportional to the percentage of students who have not yet heard the rumour, i.e., $\frac{dS}{dt} = k(100 - S)$. Initially, no students have heard the rumour, but after just 1 day, 10% of students have heard the rumour.
- (a) (8 points) Find the particular solution to the differential equation (i.e. solve the differential equation and determine all the constants).

- (b) (2 points) After how many days have 90% of students heard the rumour?

END OF EXAM

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