

# SAMPLE

---

Math 16C Sec 2 (Malkin)  
Mid-term exam 1  
Wed April 30th 2008

Name:  
Student ID:  
Signature:

**DO NOT TURN OVER THIS PAGE  
UNTIL INSTRUCTED TO DO SO!**

**Write your name, student ID, and signature NOW!**

**NO NOTES, CALCULATORS, OR BOOKS ARE ALLOWED.  
NO ASSISTANCE FROM CLASSMATES IS ALLOWED.**

Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. Be organized and neat, and use notation appropriately. You will be graded on the proper use of derivative and integral notation. Put units on answers where appropriate.

*Please write legibly!!*

#	Student's Score	Maximum possible Score
1		15
2		13
3		12
4		10
Total points		50

# SAMPLE

---

1. (a) (3 points) Show that the equation  $x^2 + xy = C$  ( $C$  is a constant) is a solution of the differential equation  $x^2y'' - 2(x + y) = 0$ .

(b) (6 points) Use any method to find the general solution of the differential equation  $y^2y' - xe^x = 0$ . You must write  $y$  as a function of  $x$ .

(c) (6 points) Use any method to find the general solution of the differential equation  $xy' + 2y = x^2 + \frac{1}{x}$ . You must write  $y$  as a function of  $x$ .

# SAMPLE

---

2. (a) (3 points) Find an equation for the sphere that has a diameter with endpoints  $(0,0,0)$  and  $(1,2,-2)$ .

- (b) (5 points) Consider the function  $f(x, y) = \ln(xy)$ . Find the *domain* and *range* of  $f$ . *Sketch* the domain.

- (c) (5 points) Consider the surface  $z = x^2 + y^2 - 1$ . Graph *separately* the intersection of the surface with the planes  $x = 0$ ,  $y = 0$ , and  $z = 0$  (i.e. graph the  $zy$ -trace, the  $xz$ -trace, and the  $xy$ -trace). Don't forget to label the axes.

# SAMPLE

---

3. (a) (2 points) Find the slope of the surface  $z = f(x, y) = \sqrt{x^2 - 2y^2}$  at the point  $(1, 0, 1)$  in the  $x$ -direction and in the  $y$ -direction.

(b) (5 points) Find all the critical points of  $f(x, y) = 2x^2y - 2x^2 - y^2 + 3$ .  
(Hint: there are three critical points).

(c) (5 points) The function  $f(x, y) = x^3 - 3x + 2y^2 - 8y$  has two critical points:  $(1, 2)$  and  $(-1, -2)$ . Classify each of the critical points of  $f$ .

# SAMPLE

---

4. (10 points) During a chemical reaction, substance  $A$  is converted into substance  $B$ . Let  $y$  be the amount of substance  $A$  after  $t$  hours. The rate at which  $A$  changes to  $B$  is proportional to square of the amount of  $A$ , i.e.  $\frac{dy}{dt} = ky^2$ . Initially, there is 100 grams of  $A$  present, and after one hour there is only 10 grams remaining.
- (a) (8 points) Find the particular solution to the differential equation (i.e. solve the differential equation and determine all the constants).

- (b) (2 points) After how many hours is there only 1 gram of  $A$  remaining?

END OF EXAM

# SAMPLE

---

This page is blank.