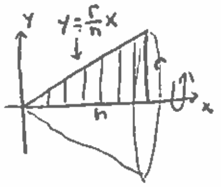
MATH 178 — SEC. 6.3 – SOLUTIONS

(32b) $\bar{V} = \frac{1}{6-0}\int_0^6 \left(-(T-3)^2+5\right)dT = \frac{1}{6}\int_0^6 \left(-(T^2-6T+9)+5\right)dT = \frac{1}{6}\int_0^6 \left(-T^2+6T-4\right)dT$

$= \frac{1}{6}\left[-\frac{T^3}{3}+3T^2-4T\right]_0^6 = \frac{1}{6}\left(-2\cdot 36+3\cdot 36-24\right) = \frac{1}{6}(36-24) = \boxed{2}$

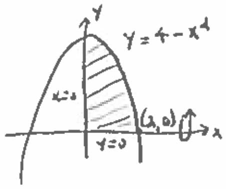
(33)



$V = \int_0^h \pi\left(\frac{r}{h}x\right)^2 dx = \int_0^h \pi\cdot\frac{r^2}{h^2}x^2\,dx = \frac{\pi r^2}{h^2}\int_0^h x^2\,dx = \frac{\pi r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h$

$= \frac{\pi r^2}{h^2}\cdot\frac{h^3}{3} = \boxed{\frac{1}{3}\pi r^2 h}$

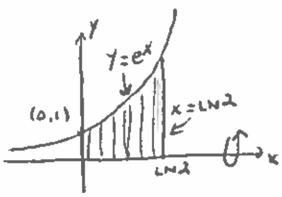
(35)



$V = \int_0^2 \pi(4-x^2)^2 dx = \pi\int_0^2 (16-8x^2+x^4)\,dx = \pi\left[16x-\frac{8}{3}x^3+\frac{x^5}{5}\right]_0^2$

$= \pi\left(32-\frac{64}{3}+\frac{32}{5}\right) = 32\pi\left(1-\frac{2}{3}+\frac{1}{5}\right) = 32\pi\left(\frac{8}{15}\right) = \boxed{\frac{256\pi}{15}}$

(38)

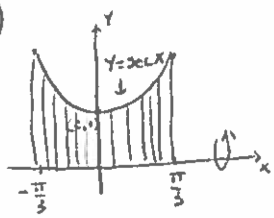


$V = \int_0^{\ln 2} \pi(e^x)^2 dx = \pi\int_0^{\ln 2} e^{2x}\,dx = \pi\left[\frac{1}{2}e^{2x}\right]_0^{\ln 2}$

$= \frac{\pi}{2}\left(e^{2\ln 2}-e^0\right) = \frac{\pi}{2}\left((e^{\ln 2})^2-1\right) = \frac{\pi}{2}(2^2-1) = \boxed{\frac{3\pi}{2}}$

(using symmetry)

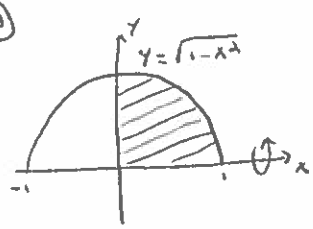
(39)



$V = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \pi(\sec x)^2 dx = 2\int_0^{\pi/3} \pi(\sec^2 x)\,dx = 2\pi\left[\tan x\right]_0^{\pi/3}$

$= 2\pi\left(\tan\frac{\pi}{3}-\tan 0\right) = 2\pi(\sqrt{3}-0) = \boxed{2\pi\sqrt{3}}$

(40)

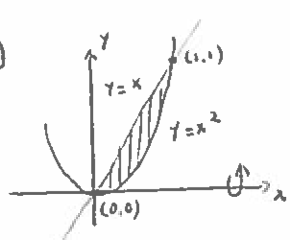


$V = \int_0^1 \pi\left(\sqrt{1-x^2}\right)^2 dx = \pi\int_0^1 (1-x^2)\,dx = \pi\left[x-\frac{x^3}{3}\right]_0^1$

$= \pi\left(1-\frac{1}{3}\right) = \boxed{\frac{2}{3}\pi}$

[CHECK: The solid is half of a solid sphere with radius 1, so $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi\cdot 1^3 = \frac{2}{3}\pi$]

(41)



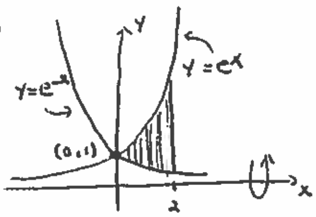
$V = \int_0^1 \pi\left(x^2-(x^2)^2\right)dx = \pi\int_0^1 (x^2-x^4)\,dx = \pi\left[\frac{x^3}{3}-\frac{x^5}{5}\right]_0^1$

$= \pi\left(\frac{1}{3}-\frac{1}{5}\right) = \boxed{\frac{2\pi}{15}}$

MATH 16B

Sec. 6.3 – SOLUTIONS

(43)

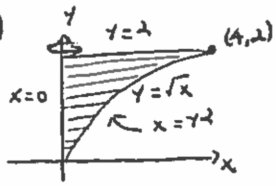


$$V = \int_0^2 \pi\left((e^x)^2 - (e^{-x})^2\right) dx = \pi \int_0^2 (e^{2x} - e^{-2x})\, dx$$

$$= \pi\left[\tfrac{1}{2}e^{2x} - \left(-\tfrac{1}{2}\right)e^{-2x}\right]_0^2 = \frac{\pi}{2}\left((e^4 + e^{-4} - (e^0 + e^0)\right)$$

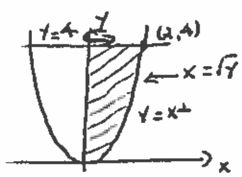
$$= \boxed{\frac{\pi}{2}\left(e^4 + e^{-4} - 2\right)}$$

(47)



$$V = \int_0^2 \pi (y^2)^2\, dy = \pi \int_0^2 y^4\, dy = \pi\left[\frac{y^5}{5}\right]_0^2 = \boxed{\frac{32\pi}{5}}$$

(48)



$$V = \int_0^4 \pi (\sqrt{y})^2\, dy = \pi \int_0^4 y\, dy = \pi\left[\frac{y^2}{2}\right]_0^4 = \boxed{8\pi}$$