MATH 17B ANSWERS – PINK SHEET

1) $\int x^3 e^{5x}\,dx$ INTEGRATE BY PARTS (3 TIMES) LET $u = x^3$, $dv = e^{5x}dx$
$du = 3x^2\,dx$, $v = \frac{1}{5}e^{5x}$
(TABULAR INTEGRATION WORKS WELL)

2) $\int \frac{x^4}{x^2-5}\,dx$ PARTIAL FRACTIONS

$$\begin{array}{r|l} & x^2+5 \\ x^2-5 & x^4 \\ & x^4-5x^2 \\ & 5x^2 \\ & 5x^2-25 \\ & 25 \end{array}$$

$= \int \left(x^2+5+\frac{25}{x^2-5}\right)dx$

$= \frac{x^3}{3} + 5x + \int \frac{25}{x^2-5}\,dx$ NOW USE

$$\frac{25}{(x-\sqrt{5})(x+\sqrt{5})} = \frac{A}{x-\sqrt{5}} + \frac{B}{x+\sqrt{5}}$$

(NOW SOLVE FOR A AND B)

3) $\int x^2 e^{5x^3}dx$ SUBSTITUTION LET $u = 5x^3$, $du = 15x^2\,dx$ TO GET

$\frac{1}{15}\int e^u\,du = \frac{1}{15}e^u + C = \frac{1}{15}e^{5x^3} + C$

4) $\int \frac{\ln x}{x^4}\,dx$ INTEGRATE BY PARTS LET $u = \ln x$, $dv = x^{-4}dx$
$du = \frac{1}{x}dx$, $v = -\frac{1}{3}x^{-3}$

5) $\int \frac{\sqrt{x}}{x-9}\,dx$ SUBSTITUTION LET $u=\sqrt{x}$, $x = u^2$, $dx = 2u\,du$ TO GET

$\int \frac{u}{u^2-9}\cdot 2u\,du = 2\int \frac{u^2}{u^2-9}\,du$ NOW USE PARTIAL FRACTIONS. (DIVIDE FIRST.)

6) $\int \frac{x^2}{(x^2+4)^2}\,dx$ INTEGRATE BY PARTS LET $u = x$, $dv = \frac{x}{(x^2+4)^2}dx$
$du = dx$, $v = -\frac{1}{2}\cdot\frac{1}{x^2+4}$

7) $\int \frac{x^5-3x+8}{x^3+3x^2}\,dx$ PARTIAL FRACTIONS

$$\begin{array}{r|l} & x^2-3x+9 \\ x^3+3x^2 & x^5 \qquad\qquad -3x+8 \\ & x^5+3x^4 \\ & -3x^4 \\ & -3x^4-9x^3 \\ & 9x^3 \\ & 9x^3+27x^2 \\ & -27x^2-3x+8 \end{array}$$

$= \int \left(x^2-3x+9+\frac{-27x^2-3x+8}{x^3+3x^2}\right)dx$

$= \frac{x^3}{3} - \frac{3}{2}x^2 + 9x + \int \frac{-27x^2-3x+8}{x^2(x+3)}\,dx$

NOW USE $\longrightarrow$ $\frac{-27x^2-3x+8}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

8) $\int x^2\sin^{-1}x\,dx$ INTEGRATE BY PARTS LET $u = \sin^{-1}x$, $dv = x^2dx$
$du = \frac{1}{\sqrt{1-x^2}}dx$, $v = \frac{x^3}{3}$

9) $\int \frac{1}{e^{2x}+5e^x}\,dx$ SUBSTITUTION LET $u = e^x$, $du = e^x\,dx$ TO GET

$\int \frac{1}{e^{2x}+5e^x}\cdot\frac{e^x}{e^x}\,dx = \int \frac{1}{u^2+5u}\cdot\frac{1}{u}\,du = \int \frac{1}{u^2(u+5)}\,du$

(OR USE $x = \ln u$ SO $dx = \frac{1}{u}du$)

NOW USE PARTIAL FRACTIONS.

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10) $\int x^5 e^{x^3}\,dx$ SUBSTITUTION LET $T = x^3$, $dT = 3x^2\,dx$ TO GET

$\frac{1}{3}\int T e^T\,dT$. NOW USE INTEGRATION BY PARTS.

$\boxed{\text{OR}}$ INTEGRATE BY PARTS : LET $u = x^3$, $dv = x^2 e^{x^3}\,dx$
$du = 3x^2\,dx$, $v = \frac{1}{3}e^{x^3}$

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11) $\int x^8 (\ln x)^2\,dx$ INTEGRATE BY PARTS (TWICE) LET $u = (\ln x)^2$, $dv = x^8\,dx$
$du = 2\ln x \cdot \frac{1}{x}\,dx$, $v = \frac{x^9}{9}$

12) $\int \frac{x+5}{x^4+9x^2}\,dx$ PARTIAL FRACTIONS $\frac{x+5}{x^4+9x^2} = \frac{x+5}{x^2(x^2+9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}$

(NOW SOLVE FOR A, B, C, D)

13) $\int x^2 \tan^{-1} x\,dx$ INTEGRATE BY PARTS LET $u = \tan^{-1} x$, $dv = x^2\,dx$
$du = \frac{1}{x^2+1}\,dx$, $v = \frac{x^3}{3}$

14) $\int \frac{1}{x\sqrt{x+4}}\,dx$ SUBSTITUTION LET $u = \sqrt{x+4}$, $x = u^2-4$, $dx = 2u\,du$ TO GET

$\int \frac{1}{(u^2-4)\,u}\cdot 2u\,du = 2\int \frac{1}{u^2-4}\,du$ NOW USE PARTIAL FRACTIONS.

15) $\int e^{2x}\cos 4x\,dx$ INTEGRATE BY PARTS (TWICE)
USE $u = e^{2x}$, $dv = \cos 4x\,dx$ OR $u = \cos 4x$, $dv = e^{2x}\,dx$
$du = 2e^{2x}\,dx$, $v = \frac{1}{4}\sin 4x$ $du = -4\sin 4x\,dx$, $v = \frac{1}{2}e^{2x}$

(CAN ALSO USE TABULAR INTEGRATION)

16) $\int \frac{\cos\theta}{\sin^2\theta + 9}\,d\theta$ SUBSTITUTION LET $u = \sin\theta$, $du = \cos\theta\,d\theta$

$= \int \frac{1}{u^2+9}\,du = \frac{1}{3}\tan^{-1}\frac{u}{3} + C = \frac{1}{3}\tan^{-1}\left(\frac{\sin\theta}{3}\right) + C$

17) $\int \frac{1}{x^3-2x^2}\,dx$ PARTIAL FRACTIONS

$\frac{1}{x^3-2x^2} = \frac{1}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

(NOW SOLVE FOR A, B, C)

REMARK #6 CAN ALSO BE FOUND BY USING $x = 2\tan\theta$, $dx = 2\sec^2\theta\,d\theta$

SO $x^2+4 = 4\tan^2\theta + 4 = 4(\tan^2\theta + 1) = 4\sec^2\theta$
