MATH 17B Sec. 7.5 – NUMERICAL INTEGRATION

TO ESTIMATE $\int_a^b f(x)\,dx$, LET $P = \{x_0, x_1, x_2, \cdots, x_n\}$ BE A PARTITION OF $[a,b]$ INTO n EQUAL SUBINTERVALS, EACH OF LENGTH $\Delta x = \frac{b-a}{n}$.

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① MIDPOINT RULE

$$\int_a^b f(x)\,dx \approx \boxed{\sum_{i=1}^{n} f(c_i)\,\Delta x}, \quad \text{WHERE } c_i = \frac{x_{i-1} + x_i}{2} \quad \text{FOR } 1 \le i \le n.$$

BOUND ON ERROR:

$$|E| \le \frac{(b-a)^3}{24n^2}\left[\max_{a \le x \le b} |f''(x)|\right]$$

② TRAPEZOIDAL RULE

$$\int_a^b f(x)\,dx \approx \boxed{\frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]}$$

COEFFICIENTS: $1, 2, 2, \cdots, 2, 1$

BOUND ON ERROR:

$$|E| \le \frac{(b-a)^3}{12n^2}\left[\max_{a \le x \le b} |f''(x)|\right]$$

③ SIMPSON'S RULE

$$\int_a^b f(x)\,dx \approx \boxed{\frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]}$$

COEFFICIENTS: $1, 4, 2, 4, 2, \cdots, 4, 2, 4, 1$

BOUND ON ERROR:

$$|E| \le \frac{(b-a)^5}{180n^4}\left[\max_{a \le x \le b} |f^{(4)}(x)|\right]$$
