MATH 21C SOLUTIONS – SEC. 14.5 AND 14.6

14.5 – (36) a) $|\vec{\nabla} f(P)| = 2\sqrt{3}$, AND $\dfrac{\vec{\nabla} f(P)}{|\vec{\nabla} f(P)|} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{\langle 1,1,-1\rangle}{\sqrt{3}}$;

so $\vec{\nabla} f(P) = 2\sqrt{3}\left(\dfrac{\langle 1,1,-1\rangle}{\sqrt{3}}\right) = \boxed{\langle 2,2,-2\rangle}$

b) IF $\vec{w} = \langle 1,1,0\rangle$, LET $\vec{u} = \dfrac{\vec{w}}{|\vec{w}|} = \dfrac{\langle 1,1,0\rangle}{\sqrt{2}}$.

THEN $D_{\vec{u}} f(P) = \vec{\nabla} f(P)\cdot\vec{u} = \boxed{\dfrac{4}{\sqrt{2}}} = \boxed{2\sqrt{2}}$

14.6 – (4a) $\underbrace{x^2 + 2xy - y^2 + z^2}_{f(x,y,z)} = 7$, $P(1,-1,3)$

$\vec{\nabla} f = \langle 2x+2y,\ 2x-2y,\ 2z\rangle$, so $\vec{n} = \vec{\nabla} f(P) = \langle 0,4,6\rangle$.

$0(x-1) + 4(y+1) + 6(z-3) = 0$ GIVES $4y + 6z = 14$ OR $\boxed{2y + 3z = 7}$

(8a) $\underbrace{x^2 + y^2 - 2xy - x + 3y - z}_{f(x,y,z)} = -4$, $P(2,-3,18)$

$\vec{\nabla} f = \langle 2x-2y-1,\ 2y-2x+3,\ -1\rangle$, so $\vec{n} = \vec{\nabla} f(P) = \langle 9,-7,-1\rangle$.

$9(x-2) - 7(y+3) - (z-18) = 0$ GIVES $\boxed{9x - 7y - z = 21}$

(12) $z = \underbrace{4x^2 + y^2}_{f(x,y)}$, $(1,1,5)$ $\quad \vec{n} = \langle f_x, f_y, -1\rangle\big|_{(1,1,5)} = \langle 8x, 2y, -1\rangle\big|_{(1,1,5)}$,

so $\vec{n} = \langle 8,2,-1\rangle$ AND $\boxed{8x + 2y - z = 5}$ IS AN EQUATION FOR THE TANGENT PLANE.

(14) $\underbrace{xyz}_{f(x,y,z)} = 1$, $\underbrace{x^2 + 2y^2 + 3z^2}_{g(x,y,z)} = 6$; $P(1,1,1)$

LET $\vec{n}_1 = \vec{\nabla} f(P) = \langle yz, xz, xy\rangle\big|_{(1,1,1)} = \langle 1,1,1\rangle$, AND

LET $\vec{n}_2 = \vec{\nabla} g(P) = \langle 2x, 4y, 6z\rangle\big|_{(1,1,1)} = \langle 2,4,6\rangle$

THEN $\vec{a} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 4 & 6 \end{vmatrix} = 2\vec{i} - 4\vec{j} + 2\vec{k} = 2\langle 1,-2,1\rangle$,

so THE TANGENT LINE TO THE CURVE OF INTERSECTION OF THE SURFACES IS GIVEN BY $\boxed{x = 1 + t,\ \ y = 1 - 2t,\ \ z = 1 + t}$

(26) b) $f(x,y) = (x+y+2)^2$ AT $P(1,2)$

$f_x(1,2) = 2(x+y+2)\cdot 1\big|_{(1,2)} = 10$ AND $f_y(1,2) = 2(x+y+2)\cdot 1\big|_{(1,2)} = 10$,

so $L(x,y) = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$= \boxed{25 + 10(x-1) + 10(y-2)}$

MATH 21C SOLUTIONS - Sec. 14.6

(5a) $\underbrace{\cos \pi x - x^2 y + e^{xz} + yz}_{f(x,y,z)} = 4$, $P(0,1,2)$

$\vec{\nabla f} = \langle -\pi \sin \pi x - 2xy + e^{xz}\cdot z, -x^2 + z, e^{xz}\cdot x + y \rangle$, so

$\vec{n} = \vec{\nabla f}(P) = \langle 2,2,1 \rangle$. $2x + 2y + z = d$ where $d = 2(0) + 2(1) + 2$, so $\boxed{2x + 2y + z = 4}$

(17) $\underbrace{x^3 + 3x^2y^2 + y^3 + 4xy - z^2}_{f(x,y,z)} = 0$, $\underbrace{x^2 + y^2 + z^2}_{g(x,y,z)} = 11$; $P(1,1,3)$

$\vec{\nabla f} = \langle 3x^2 + 6xy^2 + 4y, 6x^2y + 3y^2 + 4x, -2z \rangle$, so let $\vec{n_1} = \vec{\nabla f}(P) = \langle 13, 13, -6 \rangle$

$\vec{\nabla g} = \langle 2x, 2y, 2z \rangle$, so let $\vec{n_2} = \vec{\nabla g}(P) = \langle 2,2,6 \rangle = 2\langle 1,1,3 \rangle$

Then $\vec{a} = \vec{n_1} \times (\frac{1}{2}\vec{n_2}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 13 & 13 & -6 \\ 1 & 1 & 3 \end{vmatrix} = 45\vec{i} - 45\vec{j} + 0\vec{k} = 45\langle 1,-1,0 \rangle$,

so the tangent line to the curve of intersection of the surfaces at P is given by $\boxed{x = 1 + t,\ y = 1 - t,\ z = 3}$ (since $\langle 1,-1,0 \rangle$ is a direction vector for the line)

(19) $f(x,y,z) = \ln\sqrt{x^2 + y^2 + z^2} = \frac{1}{2}\ln(x^2 + y^2 + z^2)$

Estimate the change in f moving from $P(3,4,12)$ a distance $.1$ in the direction of $\vec{v} = \langle 3,6,-2 \rangle$.

Sol. 1 Let $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 3,6,-2 \rangle}{7} = \langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \rangle$ and $\vec{w} = .1\vec{u}$.

Then $df = (D_{\vec{u}} f(P))|\vec{w}|$ where $\vec{\nabla f} = \langle \frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \rangle$,

so $df = (\vec{\nabla f}(P)\cdot\vec{u})|\vec{w}| = (\langle \frac{3}{169}, \frac{4}{169}, \frac{12}{169} \rangle \cdot \langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \rangle)(.1)$

$= \frac{9}{169(7)}(.1) = \frac{9}{7(1690)} = \boxed{\frac{9}{11,830}}$

Sol. 2 Let $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \rangle$, let $\vec{w} = .1\vec{u} = \langle \frac{3}{70}, \frac{6}{70}, -\frac{2}{70} \rangle$,

and let $Q = (3 + \frac{3}{70}, 4 + \frac{6}{70}, 12 - \frac{2}{70}) \leftarrow (x,y,z)$

Then $df = f_x(3,4,12)(x-3) + f_y(3,4,12)(y-4) + f_z(3,4,12)(z-12)$

$= \frac{3}{169}(\frac{3}{70}) + \frac{4}{169}(\frac{6}{70}) + \frac{12}{169}(-\frac{2}{70}) = \boxed{\frac{9}{11,830}}$

(21) $g(x,y,z) = x + x\cos z - y\sin z + y$ moving from $P(2,-1,0)$ $.2$ units toward $P_1(0,1,2)$

$\vec{\nabla g} = \langle 1 + \cos z, -\sin z + 1, -x\sin z - y\cos z \rangle$

let $\vec{v} = \vec{PP_1} = \langle -2,2,2 \rangle = 2\langle -1,1,1 \rangle$, let $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle -1,1,1 \rangle}{\sqrt{3}} = \langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$,

let $\vec{w} = .2\vec{u} = \langle -\frac{1}{5\sqrt{3}}, \frac{1}{5\sqrt{3}}, \frac{1}{5\sqrt{3}} \rangle$, and $Q = (x,y,z) = (2 - \frac{1}{5\sqrt{3}}, -1 + \frac{1}{5\sqrt{3}}, 0 + \frac{1}{5\sqrt{3}})$.

Then $dg = g_x(2,-1,0)(x-2) + g_y(2,-1,0)(y-(-1)) + g_z(2,-1,0)(z-0)$

$= 2(-\frac{1}{5\sqrt{3}}) + 1(\frac{1}{5\sqrt{3}}) + 1(\frac{1}{5\sqrt{3}}) = \frac{-2}{5\sqrt{3}} + \frac{2}{5\sqrt{3}} = \boxed{0}$

(or use $dg = (\vec{\nabla g}(P)\cdot\vec{u})(|\vec{w}|) = (\langle 2,1,1 \rangle \cdot \langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle)(.2) = 0(.2) = \boxed{0}$)