Math 21C
Test 3

Printed Name _____
(FIRST) (LAST)

Signature _____

Please Show All Your Work, and Mark Your Answers Clearly.

No Calculators -- No Scratch Paper -- No Cell Phones

There are **4 pages** of problems. (The last problem is for extra credit.)

You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.

Simplify all numerical answers.

Please indicate clearly if you continue work on the back of a page.

If you finish the test during the last 10 minutes, please remain seated until the test papers have been collected from your row.

Be sure to stop working **immediately** when time is called; you are subject to a deduction from your test score if you do not.

① FIND AN EQUATION OF THE TANGENT PLANE TO THE SURFACE DEFINED BY $x^2 - 2xy + 3yz + z^2 = 18$ AT $P(4, 1, 2)$. (SIMPLIFY YOUR ANSWER)

9 PTS

② USE LAGRANGE MULTIPLIERS TO FIND THE MAX. AND MIN. VALUES OF $f(x, y) = 5x - 10y$ ON THE ELLIPSE $x^2 + 4y^2 = 128$.

9 PTS

③ FIND THE DIRECTIONAL DERIVATIVE OF $f(x, y, z) = 2xy + xz + 3yz$ AT $P(2, 1, 1)$ IN THE DIRECTION OF $\vec{v} = \langle 6, 3, 2 \rangle$.

9 PTS

④ FIND THE CRITICAL POINTS OF THE FUNCTION

$f(x,y) = 18xy - 3xy^2 - x^2y$. (YOU DO NOT HAVE TO CLASSIFY THE CRITICAL POINTS.)

8 PTS

⑤ FIND THE POINT ON THE CONE $z = \sqrt{2x^2 + 3y^2}$ WHICH IS CLOSEST TO $P(15, 12, 0)$ BY MINIMIZING A FUNCTION $f(x,y)$ OF 2 VARIABLES. (YOU DO NOT HAVE TO SHOW THAT YOUR ANSWER GIVES A MIN.)

8 PTS

⑥ IF $f(x,y,z) = x^2e^y - 2x\ln z$ AND $P = (2, \ln 2, 1)$, FIND

a) THE MAXIMAL DIRECTIONAL DERIVATIVE OF f AT P.

b) THE UNIT VECTOR IN THE DIRECTION WHICH GIVES THIS MAXIMUM

10 PTS

⑦ LET $f(x,y) = x^2 + 8xy + 2y^4$, SO $f_x = 2x + 8y$ AND $f_y = 8x + 8y^3$.

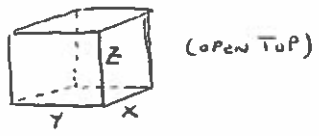
a) FIND ALL THE CRITICAL POINTS FOR f.

6 PTS

b) CLASSIFY EACH CRITICAL POINT AS CORRESPONDING TO A LOCAL MAX., LOCAL MIN., OR SADDLE POINT. (CALCULATE THE VALUE OF D AT EACH CRITICAL POINT.)

10 PTS

⑧ USE LAGRANGE MULTIPLIERS TO FIND THE MAXIMUM VOLUME FOR A RECTANGULAR BOX WITH AN OPEN TOP AND A SURFACE AREA OF 300 cm^2.



(OPEN TOP)

13 PTS

⑨ IF $h(\tau) = g(\tau^3 - 2\tau^2, \tau^2 + 8\tau)$ WHERE g IS DIFFERENTIABLE, FIND $h'(3)$ IN TERMS OF THE PARTIAL DERIVATIVES OF g.

8 PTS

⑩ IF $F(x) = \int_{x^2}^{x^5} e^{-x^2\tau^3}\, d\tau$, USE THE CHAIN RULE TO FIND AND SIMPLIFY $F'(x)$.

10 PTS

⑪ IF $f(x,y) = x^4 + 3xy^3 - 4x^3y + y^4$, SHOW WHETHER THE CRITICAL POINT $(0,0)$ CORRESPONDS TO A LOCAL MAX., A LOCAL MIN., OR A SADDLE POINT, GIVEN THAT $\underline{D=0}$ AT $(0,0)$.

10 PTS (EXTRA CREDIT)