MATH 12 SOLUTIONS – SEC. 4.4 AND 4.5

4.4 – (12) $\log_{12} 9 + \log_{12} 16 = \log_{12}(9 \cdot 16) = \log_{12} 144 = \boxed{2}$

(35) $\log_2\left(\dfrac{x(x^2+1)}{\sqrt{x^2-1}}\right) = \log_2\left(x(x^2+1)\right) - \log_2\sqrt{x^2-1}$

$= \log_2 x + \log_2(x^2+1) - \log_2(x^2-1)^{1/2}$

$= \boxed{\log_2 x + \log_2(x^2+1) - \tfrac{1}{2}\log_2(x^2-1)}$

(43) $\ln\left(\dfrac{x^3\sqrt{x-1}}{3x+4}\right) = \ln\left(x^3(x-1)^{1/2}\right) - \ln(3x+4)$

$= \ln x^3 + \ln(x-1)^{1/2} - \ln(3x+4)$

$= \boxed{3\ln x + \tfrac{1}{2}\ln(x-1) - \ln(3x+4)}$

(73) THE ERROR IS IN THE FIRST LINE: $\log .1 = -1 > -2 = 2\log .1$

4.5 – (8) $3^{2x-1} = 5$ $\quad \ln 3^{2x-1} = \ln 5$ $\quad (2x-1)\ln 3 = \ln 5$ $\quad 2x\ln 3 - \ln 3 = \ln 5$

$2x\ln 3 = \ln 3 + \ln 5 = \ln 15$ $\quad x = \boxed{\dfrac{\ln 15}{2\ln 3}}$

(11) $e^{1-4x} = 2$ $\quad \ln e^{1-4x} = \ln 2$ $\quad 1-4x = \ln 2$ $\quad 1-\ln 2 = 4x$ $\quad x = \boxed{\tfrac{1}{4}(1-\ln 2)}$

(23) $2^{3x+1} = 3^{x-2}$ $\quad \ln 2^{3x+1} = \ln 3^{x-2}$ $\quad (3x+1)\ln 2 = (x-2)\ln 3$

$3x\ln 2 + \ln 2 = x\ln 3 - 2\ln 3$ $\quad 3x\ln 2 - x\ln 3 = -\ln 2 - 2\ln 3$

$(3\ln 2 - \ln 3)x = -(\ln 2 + 2\ln 3)$ $\quad x = \boxed{-\dfrac{\ln 2 + 2\ln 3}{3\ln 2 - \ln 3}} = \boxed{\dfrac{\ln 2 + 2\ln 3}{\ln 3 - 3\ln 2}}$

(27) $100(1.04)^{2T} = 300$ $\quad (1.04)^{2T} = 3$ $\quad \ln(1.04)^{2T} = \ln 3$

$2T\ln 1.04 = \ln 3$ $\quad T = \boxed{\dfrac{\ln 3}{2\ln 1.04}}$

(30) $e^{2x} - e^x - 6 = 0$ $\quad (e^x-3)(e^x+2) = 0$ $\quad e^x = 3$ OR $\cancel{e^x = -2}$ ← (NOT POSSIBLE, SINCE $e^x > 0$)

$x = \boxed{\ln 3}$

(36) $x^2e^x + xe^x - e^x = 0$ $\quad (x^2+x-1)e^x = 0$

$x^2+x-1 = 0$ OR $\cancel{e^x = 0}$ ← (NOT POSSIBLE)

$x = \dfrac{-1 \pm\sqrt{1+4}}{2} = \boxed{\dfrac{-1\pm\sqrt{5}}{2}}$

(38) $\ln(2+x) = 1$ $\quad 2+x = e^1 = e$ $\quad x = \boxed{e-2}$

(44) $\log_2(x^2-x-2) = 2$ $\quad x^2-x-2 = 2^2 = 4$ $\quad x^2-x-6 = 0$

$(x-3)(x+2) = 0$ $\quad \boxed{x=3}$ OR $\boxed{x=-2}$

(50) $\log_3(x+15) - \log_3(x-1) = 2$ $\quad \log_3\dfrac{x+15}{x-1} = 2$ $\quad \dfrac{x+15}{x-1} = 3^2 = 9$

$x+15 = 9(x-1)$ $\quad x+15 = 9x-9$ $\quad 24 = 8x$ $\quad \boxed{x=3}$ ← (CHECKS)

MATH 12

SOLUTIONS – SEC. 4.3 – 4.5

4.3 – 95 $\log 1000 = 3$, AND 1000 HAS 4 DIGITS

$\log 10{,}000 = 4$, AND 10,000 HAS 5 DIGITS

IF $10^n \le m < 10^{n+1}$, THEN $n \le \log m < n+1$ SO $[\![\log m]\!] = n$

(WHERE $[\![x]\!]$ IS THE LARGEST INTEGER WHICH IS LESS THAN OR EQUAL TO x),

AND m HAS $n+1 = [\![\log m]\!] + 1$ DIGITS.

IN PARTICULAR, 2^{100} HAS $[\![\log 2^{100}]\!] + 1 = [\![100 \log 2]\!] + 1 = [\![30.1\ldots]\!] + 1 = 30 + 1 = \boxed{31}$ DIGITS

4.4 – 66 $(\log_2 5)(\log_5 7) = \frac{\cancel{\ln 5}}{\ln 2} \cdot \frac{\ln 7}{\cancel{\ln 5}} = \frac{\ln 7}{\ln 2} = \boxed{\log_2 7}$

4.5 – 27 $e^{2x} - 3e^x + 2 = 0$

$(e^x - 2)(e^x - 1) = 0$ $\quad e^x = 2$ OR $e^x = 1$ $\quad$ (IF $e^x = T$, THEN $x = \ln T$)

$x = \boxed{\ln 2}$ $\quad x = \boxed{0} \longleftarrow \ln 1$

37 $\ln x = 10$ $\quad \log_e x = 10$ SO $\boxed{x = e^{10}}$

OR IF $\ln x = 10$, $x = e^{\ln x} = \boxed{e^{10}}$ $\quad$ (SINCE $e^{\ln x} = x$ IF $x > 0$)

51 $\log_2 x + \log_2 (x-3) = 2$

$\log_2 (x(x-3)) = 2$

$\log_2 (x^2 - 3x) = 2$ $\quad x^2 - 3x = 2^2 = 4$ $\quad x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$ $\quad \boxed{x = 4}$ OR $\cancel{x = -1}$ $\longleftarrow$ (DOESN'T CHECK)

55 SOLVE $\log(x+3) = \log x + \log 3$.

$\log(x+3) = \log(3x)$

$x + 3 = 3x$ $\quad$ (SINCE $f(x) = \log x$ IS 1–1, OR RAISING 10 TO BOTH SIDES)

$3 = 2x$

$\boxed{x = \frac{3}{2}}$