MATH 12 SEC. 7.5 – SOLUTIONS

(43) $\sin 2\theta + \cos\theta = 0$

$2\sin\theta\cos\theta + \cos\theta = 0$ $\quad \cos\theta(2\sin\theta + 1) = 0$

$\cos\theta = 0$: $\boxed{\theta = \frac{\pi}{2}}$ OR $\boxed{\theta = \frac{3\pi}{2}}$

OR $\sin\theta = -\frac{1}{2}$: $\bar{\theta} = \frac{\pi}{6}$ $\quad \boxed{\theta = \frac{7\pi}{6}}$ (Q. III) OR $\boxed{\theta = \frac{11\pi}{6}}$ (Q. IV)

(45) $\cos 2\theta + \cos\theta = 2$

$2\cos^2\theta - 1 + \cos\theta = 2$ $\quad 2\cos^2\theta + \cos\theta - 3 = 0$ $\quad (2\cos\theta + 3)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{3}{2}$ (NO SOL.) OR $\cos\theta = 1$: $\boxed{\theta = 0}$

(47) $\cos 2\theta - \cos^2\theta = 0$

$2\cos^2\theta - 1 - \cos^2\theta = 0$ $\quad \cos^2\theta = 1$ $\quad \cos\theta = 1$: $\boxed{\theta = 0}$

OR $\cos\theta = -1$: $\boxed{\theta = \pi}$

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(49) $\cos 2\theta - \frac{1}{2}(1 + \cos 2\theta) = 0$ $\quad \cos 2\theta - \frac{1}{2} - \frac{1}{2}\cos 2\theta = 0$ $\quad \frac{1}{2}\cos 2\theta = \frac{1}{2}$

$\cos 2\theta = 1$ $\quad 2\theta = 0, 2\pi$ $\quad \boxed{\theta = 0, \pi}$

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(48) $2\sin^2\theta = 2 + \cos 2\theta$

$2\sin^2\theta = 2 + 1 - 2\sin^2\theta$ $\quad 4\sin^2\theta = 3$ $\quad \sin^2\theta = \frac{3}{4}$ $\quad \sin\theta = \pm\frac{\sqrt{3}}{2}$ $\quad \bar{\theta} = \frac{\pi}{3}$

$\theta = \boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

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(50) $2 \cdot \frac{1}{2}(1 - \cos 2\theta) = 2 + \cos 2\theta$ $\quad 1 - \cos 2\theta = 2 + \cos 2\theta$ $\quad -1 = 2\cos 2\theta$

$\cos 2\theta = -\frac{1}{2}$ $\quad$ REF. ANGLE $= \frac{\pi}{3}$ $\quad 2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$\theta = \boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

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(51) $\cos\theta - \sin\theta = \sqrt{2}\sin\frac{\theta}{2}$

$(\cos\theta - \sin\theta)^2 = (\sqrt{2}\sin\frac{\theta}{2})^2$

$\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta = 2\sin^2\frac{\theta}{2}$

$1 - 2\cos\theta\sin\theta = 2 \cdot \frac{1}{2}(1 - \cos\theta) = 1 - \cos\theta$

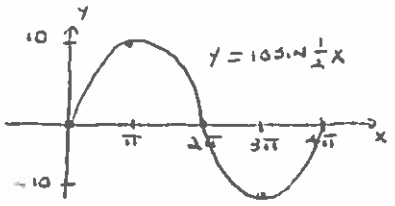
$\cos\theta - 2\cos\theta\sin\theta = 0$ $\quad$ (DOESN'T CHECK)

$\cos\theta(1 - 2\sin\theta) = 0$ $\quad \cos\theta = 0$: $\theta = \frac{\pi}{2}$ OR $\boxed{\theta = \frac{3\pi}{2}}$

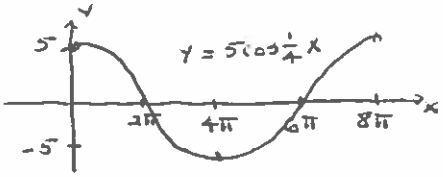
OR $\sin\theta = \frac{1}{2}$: $\boxed{\theta = \frac{\pi}{6}}$ OR $\theta = \frac{5\pi}{6}$

MATH 12 — SOLUTIONS – SEC. 5.3, 5.5, 7.5

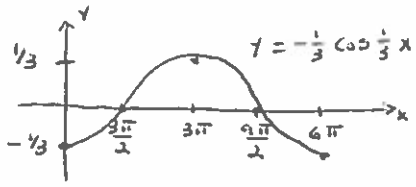
(21) $y = 10\sin\frac{1}{2}x$ AMPLITUDE = 10, PERIOD = $\frac{2\pi}{1/2} = 4\pi$



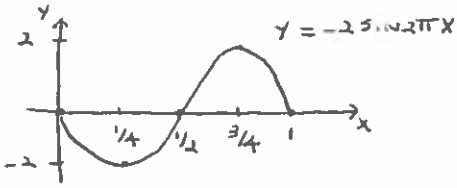
(22) $y = 5\cos\frac{1}{4}x$ AMPLITUDE = 5, PERIOD = $\frac{2\pi}{1/4} = 8\pi$



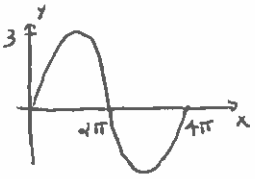
(23) $y = -\frac{1}{3}\cos\frac{1}{3}x$ AMPLITUDE = $\frac{1}{3}$, PERIOD = $\frac{2\pi}{1/3} = 6\pi$



(25) $y = -2\sin 2\pi x$ AMPLITUDE = 2, PERIOD = $\frac{2\pi}{2\pi} = 1$



(46)



AMPLITUDE = 3, PERIOD = 4π

PHASE SHIFT = 0

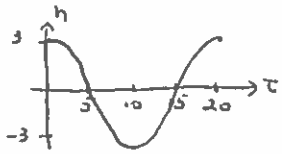
$y = 3\sin Bx$ WHERE $\frac{2\pi}{B} = 4\pi$ SO $2\pi = 4\pi B$ AND $B = 1/2$:

$\boxed{y = 3\sin\frac{1}{2}x = 3\sin\frac{x}{2}}$

(47) $h(t) = 3\cos\left(\frac{\pi}{10}t\right)$

a) PERIOD = $\frac{2\pi}{B} = \frac{2\pi}{\pi/10} = 2\cancel{\pi}\cdot\frac{10}{\cancel{\pi}} = \boxed{20 \text{ sec}}$

b) AMPLITUDE = 3, SO WAVE HEIGHT = $\boxed{6 \text{ FT}}$

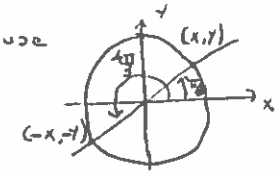


5.5 – (29) $\cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \boxed{\frac{5\pi}{6}}$ SINCE $0 \le \frac{5\pi}{6} \le \pi$

(eg use $\cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$)

(36) $\tan^{-1}\left(\tan\frac{4\pi}{3}\right) = \tan^{-1}\sqrt{3} = \boxed{\frac{\pi}{3}}$

(eg use)



$\tan\frac{4\pi}{3} = \frac{-y}{-x} = \frac{y}{x} = \tan\frac{\pi}{3}$, AND $-\frac{\pi}{2} < \frac{\pi}{3} < \frac{\pi}{2}$

7.5 – (40) $\cos\theta\cos 2\theta + \sin\theta\sin 2\theta = \frac{1}{2}$, θ IN $[0, 2\pi)$

$\cos(\theta - 2\theta) = \frac{1}{2}$

$\cos(-\theta) = \frac{1}{2}$ SO $\cos\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}}$ (Q. I)

$\boxed{\theta = \frac{5\pi}{3}}$ (Q. IV)

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$2\pi - \frac{\pi}{3}$