MATH 12

ANSWERS – TEST 2 REVIEW PROBLEMS

① $e^{2x} - 3e^{x} - 10 = 0$

$(e^{x} - 5)(e^{x} + 2) = 0$ $\quad \underline{e^{x} = 5}$ OR $\underline{e^{x} = -2}$ ← (NO SOLUTION, SINCE $e^{x} > 0$ FOR ALL X)

$\boxed{X = \ln 5}$

(OR SUBSTITUTE $\underline{T = e^{x}}$ FIRST TO GET $T^{2} - 3T - 10 = 0$)

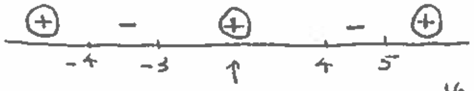
② $\ln\left(\dfrac{\sqrt[4]{5x-8}}{\sqrt{x+3}\,(2x+1)^{6}}\right) = \ln\left(\sqrt[4]{5x-8}\right) - \ln\left(\sqrt{x+3}\,(2x+1)^{6}\right)$

$= \ln(5x-8)^{1/4} - \left[\ln(x+3)^{1/2} + \ln(2x+1)^{6}\right]$

$= \boxed{\tfrac{1}{4}\ln(5x-8) - \tfrac{1}{2}\ln(x+3) - 6\ln(2x+1)}$

③ $Y = \ln\left(\dfrac{x^{2}-16}{x^{2}-2x-15}\right)$ IS DEFINED WHERE $\underline{\dfrac{x^{2}-16}{x^{2}-2x-15} > 0}$,

SO $T \rightarrow \dfrac{(x-4)(x+4)}{(x-5)(x+3)} > 0$:



IF $x = 0$, $T = \dfrac{-16}{-15} = \dfrac{16}{15}$

DOMAIN: $\boxed{(-\infty, -4) \cup (-3, 4) \cup (5, \infty)}$

④ A) $e^{2\ln 5} + \ln\dfrac{1}{e^{4}} = (e^{\ln 5})^{2} + \ln e^{-4} = 5^{2} - 4 = 25 - 4 = \boxed{21}$

B) $30\log_{6}\sqrt{6} + 5^{2\log_{5}3} = 30\log_{6}6^{1/2} + (5^{\log_{5}3})^{2} = 30\left(\tfrac{1}{2}\right) + 3^{2} = 15 + 9 = \boxed{24}$

C) $\log_{16}96 - \log_{16}3 = \log_{16}\dfrac{96}{3} = \log_{16}32 = \dfrac{\log_{2}32}{\log_{2}16} = \boxed{\dfrac{5}{4}}$

(OR LET $\underline{T = \log_{16}32}$, SO $16^{T} = 32$, $(2^{4})^{T} = 2^{5}$, $2^{4T} = 2^{5}$, $4T = 5$, $T = \boxed{\dfrac{5}{4}}$)

⑤ A) $\log_{3}(x+3) + \log_{3}(x-5) = 2$

$\log_{3}((x+3)(x-5)) = 2$ $\quad \log_{3}(x^{2}-2x-15) = 2$ $\quad x^{2}-2x-15 = 3^{2} = 9$

$x^{2} - 2x - 24 = 0$ $\quad (x-6)(x+4) = 0$ $\quad \boxed{X = 6}$ OR $\cancel{X = -4}$ ← (DOESN'T CHECK)

B) $5x(\ln x)^{2} - 2x\,\underline{\ln(x^{5})} = 0$

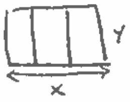
$5x(\ln x)^{2} - 2x \cdot \underline{5\ln x} = 0$

$5x\ln x(\ln x - 2) = 0$

$\underline{\cancel{x = 0}}$ OR $\underline{\ln x = 0}$ OR $\underline{\ln x = 2}$

↑ (DOESN'T CHECK) $\quad \boxed{X = 1}$ $\quad \boxed{X = e^{2}}$

⑥



1) $\underline{A = XY}$

2) $\underline{F = 2X + 4Y = 200}$ SO $X + 2Y = 100$ AND $\underline{X = 100 - 2Y}$

3) $\underline{A} = (100 - 2Y)Y = \underline{100Y - 2Y^{2}}$

4) A HAS A MAX. WHEN $\underline{Y} = -\dfrac{100}{2(-2)} = \boxed{25\text{M}}$

AND $\underline{X} = 100 - 2(25) = \boxed{50\text{M}}$

(OR USE $\underline{Y = 50 - \tfrac{1}{2}X}$ TO GET $\underline{A = 50X - \tfrac{1}{2}X^{2}}$, WHICH HAS A MAX. WHEN $\underline{X} = -\dfrac{50}{2(-\frac{1}{2})} = \boxed{50\text{M}}$ AND $\underline{Y} = 50 - \tfrac{1}{2}(50) = \boxed{25\text{M}}$)

⑦ $y = \dfrac{x^2-5x-14}{x-8}$

$$\begin{array}{r} x+3 \\ x-8\,\overline{)\,x^2-5x-14} \\ \underline{x^2-8x} \\ 3x-14 \\ \underline{3x-24} \\ 10 \end{array}$$

$= x+3 + \dfrac{10}{x-8}$

SLANTED ASYMPTOTE

$\boxed{y = x+3}$

⑧ $f(x) = e^{5x} - 4$

1) $\underline{y = e^{5x} - 4}$ 2) $y + 4 = e^{5x}$ $\ln(y+4) = \ln e^{5x} = 5x$ $\underline{x = \frac{1}{5}\ln(y+4)}$

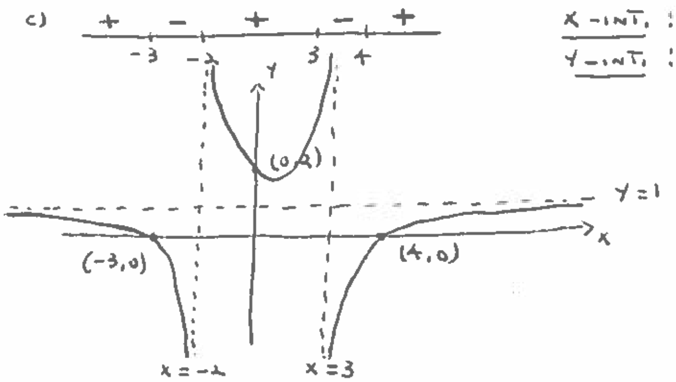
3) $\boxed{f^{-1}(y) = \frac{1}{5}\ln(y+4)}$

⑨ $y = \dfrac{(x-4)(x+3)}{(x-3)(x+2)} = \dfrac{x^2-x-12}{x^2-x-6}$

A) VERTICAL: $\underline{x=3,\ x=-2}$ HORIZONTAL: $\underline{y=1}$

B) $\dfrac{x^2-x-12}{x^2-x-6} = 1$ $x^2-x-12 = x^2-x-6$ $-12 = -6$ NO SOLUTION

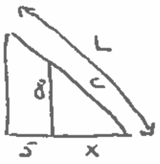
C)



X-INT.: Y=0 GIVES $x = \underline{4}$, $x = \underline{-3}$

Y-INT.: X=0 GIVES $y = \underline{2}$

⑩

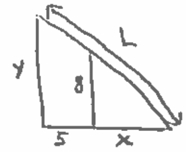


1) $\dfrac{L}{x+5} = \dfrac{C}{x}$, SO $\underline{L = \frac{C}{x}(x+5)}$

2) $C^2 = x^2 + 8^2 = x^2 + 64$, SO $\underline{C = \sqrt{x^2+64}}$

3) $\boxed{L = \frac{\sqrt{x^2+64}}{x}(x+5)} = \boxed{\sqrt{x^2+64}\left(\frac{x+5}{x}\right)}$

10A



1) $L^2 = (x+5)^2 + y^2$, SO $\underline{L = \sqrt{(x+5)^2 + y^2}}$

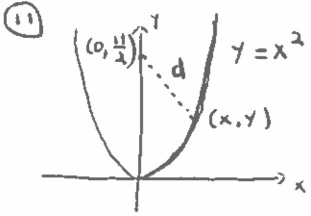
2) $\dfrac{y}{8} = \dfrac{x+5}{x}$, SO $\underline{y = \dfrac{8(x+5)}{x}}$

3) $\underline{L} = \sqrt{(x+5)^2 + \dfrac{64(x+5)^2}{x^2}} = \sqrt{(x+5)^2\left(1 + \dfrac{64}{x^2}\right)}$

$= \sqrt{(x+5)^2}\sqrt{1+\dfrac{64}{x^2}} = |x+5|\sqrt{1+\dfrac{64}{x^2}} = \boxed{(x+5)\sqrt{1+\dfrac{64}{x^2}}}$

(SINCE $x+5>0$,
SO $|x+5| = x+5$)

⑪



1) minimize $d = \sqrt{(x-0)^2 + (Y - \frac{11}{2})^2}$

or $d^2 = x^2 + (Y - \frac{11}{2})^2$

2) $Y = x^2$, so

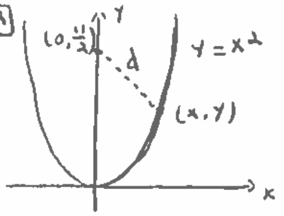
3) $d^2 = x^2 + (x^2 - \frac{11}{2})^2 = x^2 + (x^4 - 11x^2 + \frac{121}{4}) = x^4 - 10x^2 + \frac{121}{4}$

4) Let $\boxed{T = x^2}$, so $d^2 = T^2 - 10T + \frac{121}{4}$

d^2 has a min. when $T = -\frac{-10}{2(1)} = 5$, so $x^2 = 5$ and $\boxed{x = \sqrt{5}}$ (since $x > 0$ on the right half)

and $\boxed{Y = 5}$ (since $Y = x^2$)

OR



1) minimize $d = \sqrt{(x-0)^2 + (Y - \frac{11}{2})^2}$

or $d^2 = x^2 + (Y - \frac{11}{2})^2$

2) since $Y = x^2$,

3) $d^2 = Y + (Y - \frac{11}{2})^2 = Y + (Y^2 - 11Y + \frac{121}{4}) = Y^2 - 10Y + \frac{121}{4}$

4) d^2 has a min. when $Y = -\frac{-10}{2(1)} = \boxed{5}$ and $\boxed{x = \sqrt{5}}$ (since $x = \sqrt{Y}$ on the right half of $Y = x^2$).

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⑫ $e^x - 2e^{-x} = 4$ $\quad e^x - \frac{2}{e^x} = 4$ mult. by e^x to get

$e^x(e^x - \frac{2}{e^x}) = e^x(4)$, $\quad e^{2x} - 2 = 4e^x$,

$e^{2x} - 4e^x - 2 = 0$ $\quad$ Let $\boxed{T = e^x}$

$T^2 - 4T - 2 = 0$

$T^2 - 4T + 4 = 2 + 4$

$(T-2)^2 = 6$ $\quad T - 2 = \pm\sqrt{6}$ $\quad T = 2 \pm \sqrt{6}$

so $e^x = 2 + \sqrt{6}$ or $e^x = 2 - \sqrt{6}$ ← (No solution, since $e^x > 0$ for all x)

$\boxed{x = \ln(2 + \sqrt{6})}$