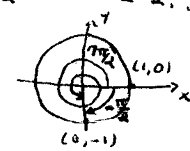
① $\cos\left(x+\frac{3\pi}{2}\right) = \cos x \cos\frac{3\pi}{2} - \sin x \sin\frac{3\pi}{2} = \cos x \cdot 0 - \sin x\,(-1) = \boxed{\sin x}$

② $\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

③ A) $\sin^{-1}\frac{1}{2} = \underline{\frac{\pi}{6}}$ B) $\sin^{-1} 1 = \underline{\frac{\pi}{2}}$ C) $\sin^{-1}\left(\sin\frac{7\pi}{2}\right) = \underline{-\frac{\pi}{2}}$ (SINCE $\sin\frac{7\pi}{2} = \sin\left(-\frac{\pi}{2}\right)$, AND $-\frac{\pi}{2}$ IS IN $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

D) $\tan^{-1} 1 = \underline{\frac{\pi}{4}}$ E) $\tan^{-1}\sqrt{3} = \underline{\frac{\pi}{3}}$ F) $\tan\left(\arctan\frac{8}{5}\right) = \underline{\frac{8}{5}}$



④ A) $\tan\theta = -\frac{5}{12}$, $\frac{\pi}{2} < \theta < \pi$

$\underline{\sec^2\theta = \tan^2\theta + 1} = \frac{25}{144} + 1 = \frac{169}{144}$, SO $\sec\theta = -\frac{13}{12}$

AND $\underline{\cos\theta} = \frac{1}{\sec\theta} = \boxed{-\frac{12}{13}}$ (SINCE $\sec\theta < 0$ AND $\cos\theta < 0$ IN Q. II)

B) $\underline{\sin\theta} = \left(\frac{\sin\theta}{\cos\theta}\right)\cos\theta = \tan\theta\cos\theta = \left(-\frac{5}{12}\right)\left(-\frac{12}{13}\right) = \boxed{\frac{5}{13}}$

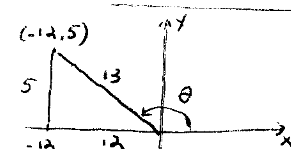
[OR USE $\underline{\sin^2\theta = 1 - \cos^2\theta} = 1 - \frac{144}{169} = \frac{25}{169}$]

C) $\underline{\cos 2\theta = \cos^2\theta - \sin^2\theta} = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

D) $\underline{\sin 2\theta = 2\sin\theta\cos\theta} = 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right) = \boxed{-\frac{120}{169}}$

[OR] $\tan\theta = \frac{Y}{X} = -\frac{5}{12}$, SO

LET $\underline{Y = 5 \text{ AND } X = -12}$



A) $\underline{\cos\theta = \frac{x}{r}} = \boxed{-\frac{12}{13}}$

B) $\underline{\sin\theta = \frac{y}{r}} = \boxed{\frac{5}{13}}$

⑤ A) $\sin 210^\circ = \boxed{-\frac{1}{2}}$ SINCE $\theta^* = 30^\circ$, $\sin 30^\circ = \frac{1}{2}$, AND $\sin\theta < 0$ IN Q. III

B) $\cos 480^\circ = \boxed{-\frac{1}{2}}$ SINCE $\cos 480^\circ = \cos 120^\circ$, $\theta^* = 60^\circ$, $\cos 60^\circ = \frac{1}{2}$, AND $\cos\theta < 0$ IN Q. II

C) $\tan\frac{47\pi}{4} = \boxed{-1}$ SINCE $\frac{47\pi}{4} \div 2\pi = \frac{47}{8} = 5 + \frac{7}{8}$, SO $\tan\frac{47\pi}{4} = \tan\left(\frac{47\pi}{4} - 5\left(\frac{8\pi}{4}\right)\right) = \tan\frac{7\pi}{4}$

WITH $\theta^* = \frac{\pi}{4}$, $\tan\frac{\pi}{4} = 1$, AND $\tan\theta < 0$ IN Q. IV

[OR $\tan\frac{47\pi}{4} = \tan\left(\frac{47\pi}{4} - 6\left(\frac{8\pi}{4}\right)\right) = \tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = \boxed{-1}$]

⑥ $\cos\theta = \frac{1}{8}$, $\frac{3\pi}{2} < \theta < 2\pi$ SO $\underline{\frac{3\pi}{4} < \frac{\theta}{2} < \pi}$ AND

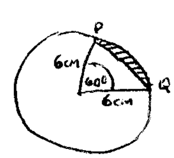
$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+1/8}{2}} = -\sqrt{\frac{9/8}{2}} = -\sqrt{\frac{9}{16}} = \boxed{-\frac{3}{4}}$ (SINCE $\cos\tau < 0$ IN Q. II)

⑦ A) $s = r\theta = 6\left(\frac{\pi}{3}\right) = \boxed{2\pi \text{ cm}}$

B) $A = A_S - A_T = \frac{1}{2}r^2\theta - \frac{1}{2}ab\sin\theta$

$= \frac{1}{2}\cdot 6^2\cdot\frac{\pi}{3} - \frac{1}{2}\cdot 6\cdot 6\sin 60^\circ$

$= 6\pi - 18\cdot\frac{\sqrt{3}}{2} = \boxed{6\pi - 9\sqrt{3} \text{ cm}^2}$



⑧ $2\cos^2\theta - 3\sin\theta - 3 = 0$

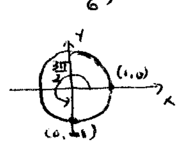
$2(1 - \sin^2\theta) - 3\sin\theta - 3 = 0$, $2 - 2\sin^2\theta - 3\sin\theta - 3 = 0$, $\underline{0 = 2\sin^2\theta + 3\sin\theta + 1}$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$, SO 1) $\underline{\sin\theta = -\frac{1}{2}}$ OR 2) $\underline{\sin\theta = -1}$

1) IF $\sin\theta = -\frac{1}{2}$, $\theta = \boxed{\frac{7\pi}{6}}$ OR $\theta = \boxed{\frac{11\pi}{6}}$ (SINCE $\theta^* = \frac{\pi}{6}$, AND $\sin\theta < 0$ IN Q. III AND Q. IV)

$\left(\pi + \frac{\pi}{6}\right)$ $\left(2\pi - \frac{\pi}{6}\right)$

2) IF $\sin\theta = -1$, $\theta = \boxed{\frac{3\pi}{2}}$



9) A) $\sqrt{x}\cdot 2(x-30) + \frac{1}{2}x^{-1/2}(x-30)^2$

$= \frac{1}{2}x^{-1/2}(x-30)\left[4x+(x-30)\right]$

$= \frac{1}{2}x^{-1/2}(x-30)\left[5x-30\right]$

$= \frac{1}{2}\cdot\frac{1}{x^{1/2}}\cdot(x-30)\cdot 5(x-6)$

$= \boxed{\dfrac{5(x-6)(x-30)}{2x^{1/2}}}$

OR $\sqrt{x}\cdot 2(x-30) + \frac{1}{2}x^{-1/2}(x-30)^2$

$= \sqrt{x}\cdot 2(x-30) + \dfrac{(x-30)^2}{2x^{1/2}} = \sqrt{x}\cdot 2(x-30)\cdot\dfrac{2x^{1/2}}{2x^{1/2}} + \dfrac{(x-30)^2}{2x^{1/2}}$

$= \dfrac{4x(x-30)+(x-30)^2}{2x^{1/2}} = \dfrac{(x-30)\left[4x+(x-30)\right]}{2x^{1/2}}$

$= \dfrac{(x-30)(5x-30)}{2\sqrt{x}} = \boxed{\dfrac{5(x-6)(x-30)}{2\sqrt{x}}}$

B) $\dfrac{(x^2+75)^2\cdot 9 - 9x\cdot 2(x^2+75)\cdot 2x}{(x^2+75)^4} = \dfrac{9\cancel{(x^2+75)}\left[x^2+75-4x^2\right]}{(x^2+75)^{\cancel{4}\,3}} = \dfrac{9\left[75-3x^2\right]}{(x^2+75)^3}$

$= \dfrac{9\cdot 3(25-x^2)}{(x^2+75)^3} = \boxed{\dfrac{27(5-x)(5+x)}{(x^2+75)^3}}$

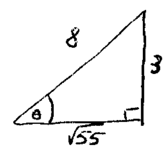
10) $x = 6\sin\theta$, $0<\theta<\frac{\pi}{2}$, so

$\dfrac{\sqrt{36-x^2}}{x} = \dfrac{\sqrt{36-(6\sin\theta)^2}}{6\sin\theta} = \dfrac{\sqrt{36-36\sin^2\theta}}{6\sin\theta} = \dfrac{\sqrt{36(1-\sin^2\theta)}}{6\sin\theta} = \dfrac{\sqrt{36\cos^2\theta}}{6\sin\theta} = \dfrac{\cancel{6}|\cos\theta|}{\cancel{6}\sin\theta}$

$= \dfrac{\cos\theta}{\sin\theta} = \boxed{\cot\theta}$ (USING THAT $\cos\theta > 0$ IN Q. I)

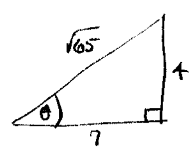
11) A) $\cos\left(\sin^{-1}\frac{3}{8}\right)$ LET $\theta = \sin^{-1}\frac{3}{8}$, SO $\sin\theta = \frac{3}{8}$

$= \cos\theta = \boxed{\dfrac{\sqrt{55}}{8}}$



B) $\sin\left(2\tan^{-1}\frac{4}{7}\right)$ LET $\theta = \tan^{-1}\frac{4}{7}$, SO $\tan\theta = \frac{4}{7}$

$= \sin 2\theta = 2\sin\theta\cos\theta = 2\left(\dfrac{4}{\sqrt{65}}\right)\left(\dfrac{7}{\sqrt{65}}\right) = \boxed{\dfrac{56}{65}}$



12) $\sin^4 6\theta = (\sin^2 6\theta)^2 = \left(\frac{1}{2}(1-\cos 12\theta)\right)^2 = \frac{1}{4}\left(1-2\cos 12\theta + \cos^2 12\theta\right)$

$= \boxed{\frac{1}{4}\left(1-2\cos 12\theta + \frac{1}{2}(1+\cos 24\theta)\right)} = \boxed{\frac{1}{4}\left(\frac{3}{2} - 2\cos 12\theta + \frac{1}{2}\cos 24\theta\right)} = \boxed{\frac{1}{8}\left(3-4\cos 12\theta + \cos 24\theta\right)}$

13)

1) $A = \frac{1}{2}r^2\theta$

2) $P = 2r + r\theta = 44$ so $r\theta = 44-2r$ AND $\theta = \dfrac{44-2r}{r} = \dfrac{44}{r} - 2$

3) $A = \frac{1}{2}r^2\left(\frac{44}{r}-2\right) = 22r - r^2$

4) A HAS A MAX. WHEN $r = -\dfrac{22}{2(-1)} = 11$,

so $A = 22(11) - 11^2 = 11(22-11) = 11^2 = \boxed{121 \text{ in}^2}$