

MAT 16B: Practice Final Exam

This is a practice exam for the final. I recommend that you set aside 2 hours to work on this alone so as to simulate real test conditions.

Name: _____

SID: _____

Signature: _____

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
Total:	0	

1. Find the area enclosed between the curves $y = x^3$ and $y = \sin\left(\frac{\pi}{2}x\right)$.

2. Evaluate the following integrals using a trigonometric substitution.

(a) $\int \sqrt{4 - x^2} dx$

(b) $\int \frac{x}{\sqrt{1 + x^2}} dx$

3. Evaluate the following integrals using any method we have learned so far.

(a) $\int x^2 \cos x \, dx$

(b) $\int \frac{4x - 1}{(x - 1)(x + 2)} \, dx$

$$(c) \int_1^{\infty} \frac{6}{\sqrt{x+3}} dx$$

$$(d) \int_4^{10} \frac{1}{(x-4)\sqrt{x-4}} dx$$

4. Consider the experiment in which we flip a coin three times. Let X be three times the number of tails which appear.

(a) List the elements of the sample space of this experiment.

(b) Write out the probability distribution for X .

(c) Find the expected value of X .

5. (a) Let $f(x) = \frac{2}{7}e^{-2x/7}$ on $[0, \infty)$. Check that f is a probability distribution.

(b) Let $f(x) = k \ln x$ on $(0, 1]$. For what value of k is f a probability distribution?

6. Let X be a random variable whose probability density function is $f(x) = -\ln x$ over the interval $(0, 1]$.

(a) Find the expected value of X .

(b) Find the variance and standard deviation of X .

(c) Find the median of X .

7. Identify the mean, variance, and standard deviation of the following probability density functions.

(a) $f(x) = 100$ on $[\frac{99}{100}, 1]$

(b) $f(x) = \pi e^{-\pi x}$ on $[0, \infty)$

(c) $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2 + x - \frac{1}{2}}$ on $(-\infty, \infty)$