Algebra Preliminary Exam 4

1. Find seven non-isomorphic groups of order 42.

2. Let R be a ring. Show that ever R-module is projective if and only if ever R-module is injective.

3. Let V be a non-zero vector space over an infinite field $\mathbb{F}$. Show that V is not the union of any finite set of proper subspaces.

4. Let R be a principal ideal domain and I be a prime ideal in R. Prove I is maximal.

5. Let W' and W be the two 3-dimensional irreducible complex representations of S_4. Let $\oplus_{i=1}^{k} V_i^{a_i}$ be the decomposition of $W' \otimes W$ into irreducible summands. Find k.

6. Let K be a field and $\alpha \in K$, and let m, n be relatively prime positive integers. Prove that $X^{mn} - \alpha$ is irreducible in $K[X]$ if and only if $X^m - \alpha$ and $X^n - \alpha$ are irreducible in $K[X]$.