

**MAT 16C: SHORT CALCULUS
SAMPLE FINAL EXAMINATION**

DATE AND TIME: WEDNESDAY, JUNE 16, 2004. 10:30–12:30.

ROOM: 202 WELLMAN

INSTRUCTOR: M. MULASE

Name: (Last) _____ (First) _____

Student ID Number: _____ – _____

Signature: _____

Remark.

- (1) You are required to abide by the *UCD Code of Academic Conduct*.
- (2) The exam set consists of 6 pages, including the cover sheet.
- (3) **Do not de-staple the set.**
- (4) This is a closed-book exam. Do not open the textbook or lecture notes during the exam.
- (5) **No calculators are allowed in the exam.**
- (6) You have full two hours.
- (7) Use the back side of the sheet if you need scratch paper.

Scores:

Page 2: _____/7

Page 3: _____/8

Page 4: _____/9

Page 5: _____/10

Page 6: _____/6

Total: _____/40

Problem 1 (4 points). *Use Newton's method to approximate a zero of the function*

$$f(x) = x^3 + x - 1 .$$

(1) Give the formula for the second approximation x_2 (2 point).

(2) Calculate the second approximation starting with $x_1 = 1$.

Work (1 point):

Answer (1 point):

Problem 2 (3 points). *Explain why Newton's method fails for finding a zero of*

$$f(x) = -x^3 + 3x^2 - x + 1$$

if you start with $x_1 = 1$.

Problem 3 (8 points). *Let us consider the function*

$$f(x) = e^{x^2} .$$

(1) Find the Taylor polynomial of degree 6 centered at $x = 0$.

Work (2 point):

Answer (2 point): _____.

(2) Approximate the integral

$$\int_0^1 e^{x^2} dx$$

using the sixth-degree Taylor polynomial.

Work (3 point):

Answer (1 point): _____.

Problem 4 (6 points). *Find the power series for*

$$f(x) = \frac{1}{x^2 + 1}$$

centered at $x = 0$.

Work (2 points):

Answer (1 point): _____.

What is the radius of convergence of the power series?

Work (2 points):

Answer (1 point): _____.

Problem 5 (3 points). *Find the Taylor polynomial of degree 6 of the function*

$$f(x) = \frac{1}{(1 + x^2)^2}$$

centered at $x = 0$.

Work (2 points):

Answer (1 point): _____.

Problem 6 (10 points). *Find the radius of convergence of each of the following series.*

(1)
$$\sum_{n=0}^{\infty} \frac{(-3x)^n}{n!} .$$

(2)
$$\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)} .$$

(3)
$$\sum_{n=0}^{\infty} \frac{1}{2-x} .$$

(4) The power series of $\ln(1-x)$.

Problem 7 (6 points). *Examine the convergence or divergence of each of the following series. Circle the convergent series.*

$$\sum_{n=0}^{\infty} n^2$$

$$\sum_{n=1}^{\infty} \frac{1}{n^7}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

$$\sum_{n=0}^{\infty} \frac{n^{10}}{(1.5)^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$$\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^2 + 1}}$$