

- 1 (32 pts.)** Determine whether or not the following limits exist, and calculate them. If the limit does not exist as a number, state whether or not it can be written as ∞ or $-\infty$.

(a) $\lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3}$

(b) $\lim_{t \rightarrow 0} \frac{\sqrt{t+1}-1}{t}$

(c) $\lim_{x \rightarrow 1^+} \frac{2-x^2}{1-x}$

(d) $\lim_{x \rightarrow \infty} \sin x$

2 (8 pts.) Using the sandwich theorem, show that $\lim_{x \rightarrow 0} x^4(1 - \cos x) = 0$.

3 (12 pts.) Directly from the definition of a limit, show that $\lim_{x \rightarrow -3} x^2 = 9$.

4 (16 pts.) Find $\lim_{x \rightarrow \infty} \frac{1 - \cos(1/x)}{(1/2x)}$. (Hint: use the double angle formula)

5 (16 pts.) Graph the function

$$f(x) = \begin{cases} \frac{x^3 - 2x^2 + 2x - 4}{x^2 - 3x + 2} : & x \neq 1, 2 \\ 4 : & x = 1, 2 \end{cases}$$

(Hint: factor the denominator first) Find all asymptotes to the graph. At which points is this function continuous, and at which points is it discontinuous? For each discontinuity, say whether or not it is removable.

6 (8 pts.) Show that the equation $x^3 - x - 1 = 0$ has a solution in the interval $[1, 2]$.

7 (8 pts.) At t seconds after liftoff, the height of a rocket is $4t^2$ feet. How fast is the rocket climbing 10 seconds after liftoff? (Compute this from our definitions, without using derivative laws)