1 (40 pts.) Differentiate the following functions. Show your work if you wish to receive partial credit. You do not need to simplify your answers.

(a) $f(x) = \frac{3x+7}{\sqrt{x^4+1}}$.

Answer:

$$f'(x) = \frac{-3x^4 - 14x^3 + 3}{(x^4+1)^{3/2}}.$$

(b) $f(x) = \frac{x^2+1}{\tan^{-1}(x^2+1)}$.

Answer:

$$f'(x) = \frac{2x((x^4+2x^2+2)\tan^{-1}(x^2+1) - (x^2+1))}{(\tan^{-1}(x^2+1))^2(x^4+2x^2+2)}.$$

(c) $f(x) = e^{2x-1}\sqrt{x+1}$.

Answer:

$$f'(x) = \frac{e^{2x-1}(4x+5)}{2\sqrt{x+1}}.$$

(d) $f(x) = \sec^2(x + \sin x)$.

Answer:

$$f'(x) = 2(1+\cos x)\sec^2(x+\sin x)\tan(x+\sin x).$$

(e) $f(x) = (\ln x)^{10}$.

Answer:

$$f'(x) = \frac{10(\ln x)^9}{x}.$$

2 (10 pts.) Suppose $u(x)$ is a differentiable function of x, and let $f(x) = x^{u(x)}$, for $x > 0$. Find $f'(x)$, in terms of $x, u(x)$, and $u'(x)$. Hint: take the natural log of both sides and use implicit differentiation.

Answer:

$$f'(x) = x^{u(x)}(u'(x)\ln x + \frac{u(x)}{x}).$$

3 (20 pts.) Consider the parametric curve $(-\sqrt{t+1}, \sqrt{3t})$.

(a) Find the tangent line to the curve at the point corresponding to $t = 3$.

Answer: $y = -2x - 1$.

(b) Find $d^2y/(dx)^2$ at the same point.

Answer: $d^2y/(dx)^2 = -1/3$ at $t = 3$

4 (10 pts.) Find the equation of the line tangent to the curve defined implicitly by $2\sin^{-1} y = x^2$ at the point $(\sqrt{\frac{\pi}{2}}, \sqrt{\frac{1}{2}})$.

Answer: $y = \frac{\sqrt{\pi}}{2}x + \frac{2-\pi}{2\sqrt{2}}$.

5 (20 pts.) Consider the function

$$f(x) = \begin{cases} x\sin\frac{1}{x} : & x \neq 0 \\ 0 : & x = 0 \end{cases}.$$

(a) Is $f(x)$ continuous at $x = 0$? Why or why not?

Answer: Yes. $x\sin\frac{1}{x}$ is sandwiched by x and $-x$.

(b) Is $f(x)$ differentiable at $x = 0$? Why or why not? If so, what is $f'(0)$?

Answer: No. $x\sin\frac{1}{x}$ hits both $y = x$ and $y = -x$ inside $(-\delta, \delta)$ for any $\delta > 0$, so the slope of the secants hits both 1 and -1 for any δ.