1 (15 pts.) Suppose $f(x)$ is a twice differentiable function on (a,b). Is it always true that every critical point for $f(x)$ is a local extremum? (Justify your answer) What are two methods of testing whether a given critical point is a local max?

Answer: No, not every critical point is a local extremum. For instance, $f(x) = x^3$ has a critical point at $x = 0$, but not a local max or local min.

One can check for a local max at a critical point by testing whether the sign of the derivative switches from positive to negative at that point, or by testing whether the second derivative is negative at that point.

2 (20 pts.) A spherical balloon is inflated at the rate of 100π cubic feet per minute. How fast is the balloon radius increasing at the instant the radius is 5 feet? How fast is the surface area increasing?

Answer: Let r be the radius in feet, t time in minutes, V volume in cubic feet, and A surface area in square feet. Then we get $\frac{dr}{dt} = 1$ ft/min and $\frac{dA}{dt} = 40\pi$ square feet per minute.

3 (15 pts.) Calculate the following limits:

(a) $\displaystyle\lim_{t\to 0} \frac{\sin t^2}{t}$.

Answer: Using L'Hopital's rule gives 0.

(b) $\displaystyle\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x}$.

Answer: Using L'Hopital's rule gives 1.

4 (25 pts.) What are the dimensions of the lightest open-top cylindrical can that will hold 1000 cubic cm of liquid?

Answer: The minimum weight (equivalently, minimum surface area) occurs when radius = height = $10/\sqrt[3]{\pi}$.

5 (25 pts.) Let $f(x) = e^x - 2e^{-x} - 3x$. Find the critical points, and the intervals on which $f(x)$ is increasing and decreasing. Find the points of inflection, and the intervals on which the graph is concave up and concave down. Sketch the graph of $f(x)$.

Answer: Critical points are at 0 and $\ln 2$. $f(x)$ is increasing on $(-\infty, 0)$ and $(\ln 2, \infty)$ and decreasing on $(0, \ln 2)$. There is a point of inflection at $x = \frac{\ln 2}{2}$, and the graph is concave up on $(\frac{\ln 2}{2}, \infty)$ and concave down on $(-\infty, \frac{\ln 2}{2})$.

(Graph omitted)