MATH 250C: PROBLEM SET #1
DUE 4/11/2008

BRIAN OSSERMAN

Exercise 1. Suppose that H and K are subgroups of G, with K normal.

(a) Show that $HK = \{hk : h \in H, k \in K\}$ is a subgroup of G.

(b) Show that if $H \cap K = \{1\}$ and H is also normal in G, then the natural map

$$H \times K \to HK$$

is an isomorphism.

G is a **simple group** if the only normal subgroups are G and $\{1\}$. Understanding the structure of simple groups is a very important part of group theory.

Exercise 2. Let G be an infinite simple group, and H a subgroup of G having finite index. Then $G = H$.

Exercise 3. Suppose G is a finite group, and H a subgroup of index n.

(a) Use the action of G by left multiplication on the set of left cosets of H in G to show that unless $|G|$ divides $n!$, it is not possible for G to be simple.

(b) Use the Sylow theorem to conclude that if G is any finite group with $|G| = p^n m$, with $m < p$, then G cannot be simple.

Exercise 4. Show that no group of order 351 is simple.

Exercise 5. Show that if G is a group of order 105, and G has a normal Sylow 3-subgroup, then G is abelian.