

MATH 250C: PROBLEM SET #3
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Exercise 1. Let H be any group, and K a cyclic group, and suppose we have

$$\varphi_1, \varphi_2 : K \rightarrow \text{Aut}(H)$$

such that there exists $\sigma \in \text{Aut}(H)$ with

$$\varphi_1(K) = \sigma(\varphi_2(K))\sigma^{-1}.$$

For K infinite, also assume that φ_1, φ_2 are injective. Then show that

$$H \rtimes_{\varphi_1} K \cong H \rtimes_{\varphi_2} K.$$

Exercise 2. Prove that up to isomorphism, there are three groups of order 75, with only one of them non-abelian.

Exercise 3. Prove that the commutator subgroup of the free group on two generators is not finitely generated.

Exercise 4. Show that $G := \langle x, y : x^2 = y^2 = (xy)^2 = 1 \rangle$ is a group of order 4.

Exercise 5. Show that $G := \langle x, y : x^3 = y^3 = (xy)^3 = 1 \rangle$ is an infinite group. Hint: use the fact that there are infinitely many primes congruent to 1 mod 3, and show that for any such prime p , the unique non-abelian group of order $3p$ is a quotient group of G .