MATH 250C: PROBLEM SET #6
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Exercise 1. Let K/F be any algebraic extension, with $\operatorname{char} F = p$.

(a) Prove that $L = \{\alpha \in K : \alpha \text{ is separable over } F\}$ is a subfield of K containing F.

(b) Prove that no element of $K \smallsetminus L$ is separable over L.

(c) Prove that for every $\alpha \in K$, for some $n \geq 1$ we have $\alpha^{p^n} \in L$. (That is, K is obtained from F by taking pth-power roots)

The L of the above exercise is called the **separable part** of K/F. Given a finite extension K/F, define the **separable degree** $[K : F]_s$ to be the degree over F of the separable part of K/F.

Exercise 2. Show that $[K : F]_s$ is multiplicative in towers of extensions.

Exercise 3. Let F be a finite field. Prove that every function $F \to F$ can be realized by a polynomial $f(x) \in F[x]$.

Exercise 4. Prove that the only automorphism of $\mathbb{R}$ is the identity. Hint: show that the condition $x > y$ can be expressed purely algebraically.

Exercise 5. Given a field F, determine the fixed field of the automorphism of $F(x)$ given by $x \mapsto x + 1$.