MATH 256A: PROBLEM SET #6
DUE 10/12/2006

BRIAN OSSERMAN

Exercise 1. (Chevalley's theorem) Do Hartshorne, Exercise 3.19 of Chapter II.

Exercise 2. (Chow's lemma) Do Hartshorne, Exercise 4.10 of Chapter II.

Exercise 3. Show that a topological space X is Hausdorff if and only if the diagonal map $X \to X \times X$ is closed.

Exercise 4. (Extra credit) Show that a topological space X is quasi-compact if and only if for every topological space Z, the projection map $X \times Z \to Z$ is closed.

For the following exercises, let X be of finite type over $\mathbb{C}$. You may use without proof (we will give a proof of this later) the following:

Theorem 5. *Let U be a Zariski open subset of X. The Zariski closure $\bar{U} \subseteq X$ has preimage in X_{an} precisely equal to the closure (in the analytic topology) of the preimage of U.*

Exercise 6. Show that if X is disconnected in the Zariski topology, then X_{an} is disconnected in the analytic topology.

Exercise 7. Show that a subset U of X is Zariski closed if and only if it is constructible and its preimage in X_{an} is closed.

Hint: for the following two exercises, you will need to use Chevalley's theorem and Chow's lemma.

Exercise 8. Show that X is separated over $\operatorname{Spec} \mathbb{C}$ if and only if X_{an} is Hausdorff.

Exercise 9. Show that X is proper over $\operatorname{Spec} \mathbb{C}$ if and only if X_{an} is compact (i.e., quasi-compact and Hausdorff).

Exercise 10. Show that $\operatorname{Spec} \mathbb{C}$ is not universally closed. Hint: consider $Z = \operatorname{Spec} \mathbb{Z}$.