MATH 256A: PROBLEM SET #8
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Exercise 1. Suppose we are given an open cover $\{U_i\}$ of a scheme Y, and isomorphisms

$$\theta_{i,j} : \mathbb{A}^n_{U_i \cap U_j} \xrightarrow{\sim} \mathbb{A}^n_{U_i \cap U_j}$$

for all $i \neq j$ such that

(i) $\theta_{i,j} = \theta_{j,i}^{-1}$ for $i \neq j$;
(ii) $\theta_{i,j} \circ \theta_{j,k} = \theta_{i,k}$ for i, j, k distinct;
(iii) each $\theta_{i,j}$ is linear in the sense of Exercise 5.18 of Chapter II of Hartshorne.

(a) Show that this data determines a unique vector bundle X of rank n on Y, such that $\psi_i : X|_{U_i} \cong \mathbb{A}^n_{U_i}$, and the $\theta_{i,j}$ are the induced isomorphisms $\psi_i \circ \psi_j^{-1}$ after restriction to $U_i \cap U_j$.

(b) Show that conversely every vector bundle X of rank n on Y can be described by data as above.

Exercise 2. Describe $\mathcal{O}(1)$ on $\mathbb{P}^n_k$ explicitly as a line bundle in the manner of the previous exercise, following the correspondence established in Exercise 5.18 of Chapter II of Hartshorne. Use the $D_+(X_i)$ as your U_i.

Exercise 3. Do Hartshorne, Exercise 5.1 of Chapter II.

Exercise 4. Do Hartshorne, Exercise 5.4 of Chapter II.

Exercise 5. Do Hartshorne, Exercise 5.7 of Chapter II.

Exercise 6. Do Hartshorne, Exercise 5.13 of Chapter II.