

MATH 256B: PROBLEM SET #4
DUE THURSDAY, MAR. 1

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Read all of the following exercises, and do (1), (6), and any three others of them.

Exercise 1. Show by considering $\mathbb{A}_k^2 \setminus \{(0,0)\}$ that it is not true in general that if \mathcal{L} is an ample invertible sheaf on a (not necessarily proper) variety X over a field, and \mathcal{F} is coherent, then $H^i(X, \mathcal{F} \otimes \mathcal{L}^n) = 0$ for $i > 0$ and $n \gg 0$. Hint: there are (at least) two ways to do this: you may find exercises III.3.3 and III.3.4 helpful for one approach.

Exercise 2. Do Hartshorne, Exercise 5.1 of Chapter III.

Exercise 3. Do Hartshorne, Exercise 5.2 of Chapter III.

Exercise 4. Do Hartshorne, Exercise 5.3 of Chapter III.

Exercise 5. Do Hartshorne, Exercise 5.5 of Chapter III.

Exercise 6. Use free resolutions to compute the following examples of Exts:

(a) $\text{Ext}^i((x, y), k[x, y])$ over $k[x, y]$ for $i \geq 0$.

(b) $\text{Ext}^i(k[t]/(t), k[t])$ over $k[t]$ for $i \geq 0$.

Exercise 7. Do Hartshorne, Exercise 6.1 of Chapter III.

Exercise 8. Do Hartshorne, Exercise 6.2 of Chapter III.