MATH 16B:
FAKE TEST 1A

WINTER 2005

(1) Graph the following function:

$$f(x) = -5^{x+3} + 2.$$

To receive full credit, you must plot at least 3 points and indicate all vertical and (or) horizontal asymptotes.

(2) The number of goods produced per day by workers in a certain industrial park is governed by the equation:

$$y = \frac{500}{2 + 3e^{-.5t}},$$

where y represents the number of goods produced after t days at the park.
a) According to the model, how many goods will a new worker produce?
b) According to the model, how many days will it take a worker produce 200 goods? (Since you cannot use a calculator, you must give an exact answer.)
c) Find the limit of the number of goods produced as time goes to infinity, and write a sentence which interprets this result physically.

(3) (a) Condense the following expression into one logarithm:

$$\frac{1}{2}\ln(x+6) - \ln(4x) + 2\ln(2).$$

(b) Use the result you found in (a) to solve the equation:

$$\frac{1}{2}\ln(x+6) - \ln(4x) + 2\ln(2) = 0.$$

(4) Find the derivative of

$$f(x) = (\cos(2x) + 1)^{x^2-3}.$$

(5) Find a function f which satisfies

$$f''(x) = x - \frac{1}{\sqrt{x-2}},$$

$f'(2) = 4$ and $f(2) = 1/3$.

(6) Integrate the following:

$$\int \frac{6x - 15}{\sqrt[3]{x^2 - 5x + 17}} dx.$$

(7) Integrate the following:

$$\int \sin(3x) + e^{-2x} dx.$$

(8) Integrate the following:

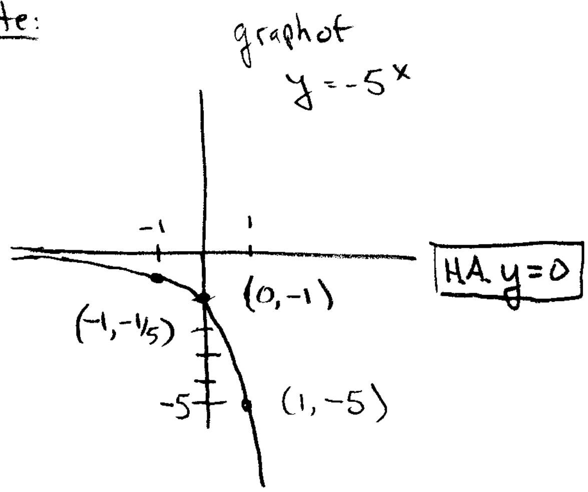
$$\int \cos^2(4x) \sin(4x) dx.$$

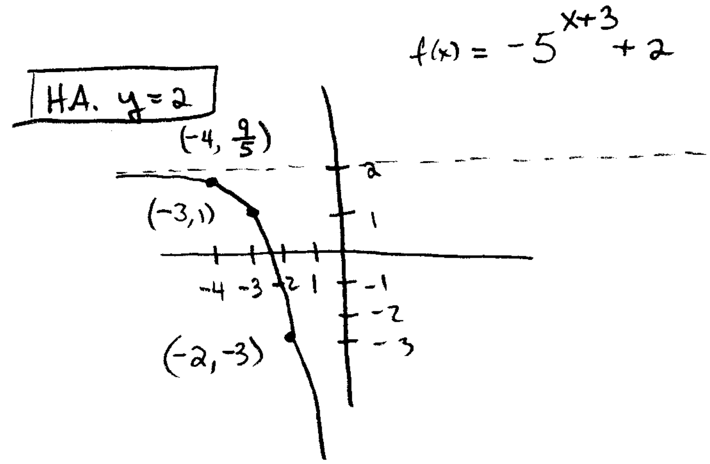
Key to Fake Test 1a:

(1) Graph $f(x) = -5^{x+3} + 2$

The graph of this function is the graph of $y = -5^x$ pushed 3 left and 2 up.

Note:





(2) $y = \dfrac{500}{2+3e^{-0.5t}}$

(a) New worker $\Rightarrow t=0$

$$y = \frac{500}{2+3} = 100 \text{ goods}$$

(b)

$$200 = \frac{500}{2+3e^{-0.5t}}$$

$$2+3e^{-0.5t} = \frac{5}{2}$$

$$3e^{-0.5t} = \frac{1}{2}$$

$$e^{-0.5t} = \frac{1}{6}$$

$$-0.5t = \ln\left(\frac{1}{6}\right)$$

$$t = \frac{-1}{0.5}\ln\left(\frac{1}{6}\right) = 2\ln(6)$$

$$= \ln(36)$$

(c) as $t \to \infty$ $\quad y \to \frac{500}{2} = 250.$

A worker that stays at the park for the rest of his/her life will eventually make 250 goods per day.

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3 (a) $\frac{1}{2}\ln(x+6) - \ln(4x) + 2\ln(2)$

$$= \ln(\sqrt{x+6}) - \ln(4x) + \ln(4)$$

$$= \ln\left(\frac{\sqrt{x+6}}{4x}\right) + \ln(4)$$

$$= \ln\left(\frac{\sqrt{x+6}}{4x} \cdot 4\right)$$

$$= \ln\left(\frac{\sqrt{x+6}}{x}\right)$$

(b) $\frac{1}{2}\ln(x+6) - \ln(4x) + 2\ln(2) = 0$

$$\ln\left(\frac{\sqrt{x+6}}{x}\right) = 0$$

$$\frac{\sqrt{x+6}}{x} = 1$$

$$\frac{x+6}{x^2} = 1$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$x = 3$ or $\cancel{x = -2}$

Not in the domain of $\ln(x)$.

4

$$f(x) = \left(\cos(2x)+1\right)^{x^2-3} = e^{(x^2-3)\ln[\cos(2x)+1]}$$

2

$$f'(x) = e^{(x^2-3)\ln[\cos(2x)+1]} \cdot \frac{d}{dx}\left[(x^2-3)\ln[\cos(2x)+1]\right]$$

$$= \left(\cos(2x)+1\right)^{x^2-3}\left[2x\cdot\ln[\cos(2x)+1] + \frac{x^2-3}{\cos(2x)+1}\cdot\frac{d}{dx}\left(\cos(2x)+1\right)\right]$$

$$= \left(\cos(2x)+1\right)^{x^2-3}\left[2x\ln[\cos(2x)+1] - \frac{2(x^2-3)}{\cos(2x)+1}\cdot\sin(2x)\right]$$

5

$$f''(x) = x - \frac{1}{\sqrt{x-2}}$$

$$f'(x) = \frac{x^2}{2} - 2\sqrt{x-2} + C$$

$$f'(2)=4 \Rightarrow 4 = f'(2) = \frac{4}{2} - 2\cancel{\sqrt{2-2}}^{\,0} + C$$

$$C = 2$$

$$f'(x) = \frac{x^2}{2} - 2\sqrt{x-2} + 2$$

$$f(x) = \frac{x^3}{6} - 2\cdot\frac{2}{3}(x-2)^{3/2} + 2x + C$$

$$\frac{1}{3} = f(2) = \frac{8}{6} - \frac{4}{3}(\cancel{2-2})^{3/2}_{\to 0} + 4 + C$$

$$C = -5$$

$$f(x) = \frac{x^3}{6} - \frac{4}{3}(x-2)^{3/2} + 2x - 5$$

⑥ $$\int \frac{6x-15}{\sqrt[3]{x^2-5x+17}}\,dx = \int \frac{3(2x-5)}{\sqrt[3]{x^2-5x+17}}\,dx$$

Let $u = x^2-5x+17$

$du = (2x-5)\,dx$

$$= 3\int u^{-1/3}\,du$$

$$= 3\,\frac{u^{-1/3+1}}{-\frac{1}{3}+1} + C$$

$$= \frac{9}{2}\left(x^2-5x+17\right)^{2/3} + C$$

7) $$\int \sin(3x) + e^{-2x}\,dx = -\frac{\cos(3x)}{3} - \frac{e^{-2x}}{2} + C$$

⑧ $$\int \cos^2(4x)\sin(4x)\,dx = \int u^2\,\frac{du}{-4}$$

Let $u = \cos(4x)$

then $du = -\sin(4x)\cdot 4\,dx$

$$= -\frac{1}{4}\,\frac{u^3}{3} + C$$

$$= -\frac{\cos^3(4x)}{12} + C$$