MATH 16B:
FAKE TEST 1B

WINTER 2005

(1) Graph the following function:

$$f(x) = \ln(x-4) + 1.$$

To receive full credit, you must plot at least 3 points and indicate all vertical and (or) horizontal asymptotes.

(2) In a research experiment, a culture of bacteria grows according to an exponential growth model. After 3 days, there are 250 bacteria, and after 5 days there are 800. Write a model which describes the growth of this culture. (i.e., find C and k) Use your results to find an explicit expression for the number of bacteria after ~~10~~ 7 days. (Since you do not have a calculator, do not try to approximate the answer.)

(3) a) Expand the following logarithm:

$$\ln\left[\frac{2\sqrt[3]{x^2-3}}{\sqrt[5]{x(x-3)}}\right].$$

(b) Find an explicit expression for x based on the equation below:

$$\frac{400}{5+3e^{-0.07x}} = 20.$$

(4) Find the derivative of

$$f(x) = (x^2 - \sqrt[3]{x} + 4)^{\sin(2x)}.$$

(5) A man kicks a ball off the top of his house which is 48 feet tall. He kicks it with an initial velocity of 32 feet per second.
a) Determine the position function, $s(t)$, describing the height of this ball t seconds after it has been kicked.
b) How long will it take the ball to hit the ground?
c) At what velocity will the ball strike the ground?

(6) Integrate the following:

$$\int \frac{3t^2 - 2t + 5}{\sqrt{t}} dt.$$

(7) Integrate the following:

$$\int \cos(2x) + \frac{3}{7 - 5x} dx.$$

(8) Integrate the following:

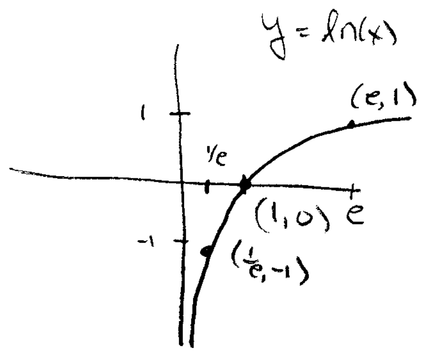
$$\int \sec^2(e^{-2x}) e^{-2x} dx.$$

Fake Test 1B

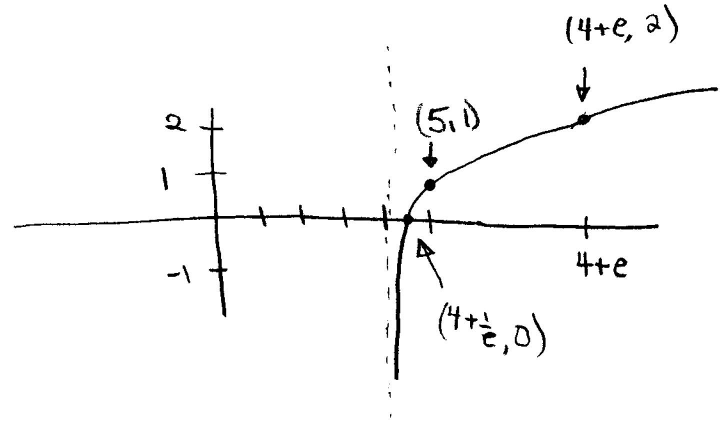
Key

① Graph $f(x) = \ln(x-4) + 1$

The graph of this function is the same as the graph of $y = \ln(x)$ pushed 4 right and 1 up.



$\boxed{\text{VA. } x=0}$



$\boxed{\text{VA. } x=4}$

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② Let $f(t) = Ce^{Kt}$ denote the number of bacteria after t days. We know

i) $f(3) = 250$ i.e. $250 = Ce^{3K}$

and

ii) $f(5) = 800$ i.e. $800 = Ce^{5K}$

Thus from i) we have that $C = \dfrac{250}{e^{3K}}$

and using ii)

$$800 = Ce^{5K} = \frac{250\,e^{5K}}{e^{3K}} = 250\,e^{2K}$$

We find that

$$e^{2k} = \frac{800}{250}$$

$$2k = \ln\left[\frac{16}{5}\right]$$

$$k = \frac{1}{2}\ln\left[\frac{16}{5}\right]$$

and so

$$C = \frac{250}{e^{3k}} = \frac{250}{e^{\frac{3}{2}\ln\left[\frac{16}{5}\right]}} = \frac{250}{\left(\frac{16}{5}\right)^{3/2}}$$

The model is given by:

$$f(t) = \frac{250}{\left(\frac{16}{5}\right)^{3/2}} e^{\frac{1}{2}\ln\left[\frac{16}{5}\right]\cdot t}$$

and so after 7 days there are

$$f(t) = \frac{250}{\left(\frac{16}{5}\right)^{3/2}} e^{\frac{7}{2}\ln\left[\frac{16}{5}\right]} = \frac{250}{\left(\frac{16}{5}\right)^{3/2}} \cdot \left(\frac{16}{5}\right)^{7/2} = 250\left(\frac{16}{5}\right)^{2}$$

$$= 10 \cdot (16)^2$$

$$= 2{,}560$$

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3) a)

$$\ln\left[\frac{2\sqrt[3]{x^2-3}}{\sqrt[5]{x(x-3)}}\right] = \ln\left[2\sqrt[3]{x^2-3}\right] - \ln\left[\sqrt[5]{x(x-3)}\right]$$

$$= \ln[2] + \frac{1}{3}\ln[x^2-3] - \frac{1}{5}\ln[x] - \frac{1}{5}\ln[x-3]$$

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③ b)

$$\frac{400}{5+3e^{-0.07x}} = 20$$

$$\frac{400}{20} = 5+3e^{-0.07x}$$

$$15 = 3e^{-0.07x}$$

$$e^{-0.07x} = 5$$

$$-0.07x = \ln(5)$$

$$x = \frac{-1}{0.07}\ln(5)$$

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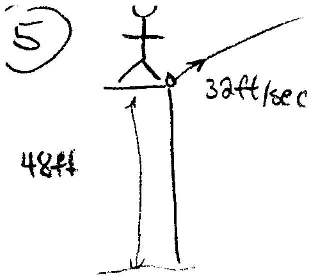
④ $$f(x) = (x^2 - \sqrt[3]{x} + 4)^{\sin(2x)} = e^{\sin(2x)\ln[x^2-\sqrt[3]{x}+4]}$$

$$f'(x) = e^{\sin(2x)\cdot\ln[x^2-\sqrt[3]{x}+4]} \cdot \frac{d}{dx}\left[\sin(2x)\cdot\ln[x^2-\sqrt[3]{x}+4]\right]$$

$$= (x^2-\sqrt[3]{x}+4)^{\sin(2x)} \cdot \left[2\cos(2x)\ln[x^2-\sqrt[3]{x}+4] + \frac{\sin(2x)}{x^2-\sqrt[3]{x}+4}\cdot\frac{d}{dx}(x^2-\sqrt[3]{x}+4)\right]$$

$$= (x^2-\sqrt[3]{x}+4)^{\sin(2x)}\left[2\cos(2x)\ln[x^2-\sqrt[3]{x}+4] + \frac{\sin(2x)}{x^2-\sqrt[3]{x}+4}\left(2x - \frac{1}{3x^{2/3}}\right)\right]$$

5)



$S''(t) = -32$

$S'(t) = -32t + C$

$S'(0) = 32 \Rightarrow C = 32$

$S'(t) = -32t + 32$

$S(t) = -16t^2 + 32t + C$

$S(0) = 48 \Rightarrow C = 48$

)

$s(t) = -16t^2 + 32t + 48$

) $s(t) = 0 \iff -16(t^2 - 2t - 3) = 0$

$(t-3)(t+1) = 0 \qquad t = 3, \cancel{-1}$

) $S'(3) = -32(3) + 32 = 32(1-3) = -64$ ft/sec

) $\int \frac{3t^2 - 2t + 5}{\sqrt{t}}\, dt = 3\int t^{3/2} dt - 2\int t^{1/2} dt + 5\int t^{-1/2} dt$

$= \frac{6}{5} t^{5/2} - \frac{4}{3} t^{3/2} + 10 t^{1/2} + C$

7) $\int \cos(2x) + \frac{3}{7-5x}\, dx = \frac{1}{2}\sin(2x) - \frac{3}{5}\ln|7-5x| + C$

8) $\int \sec^2(e^{-2x}) \cdot e^{-2x}\, dx = \int \sec^2(u) \frac{du}{-2}$

Let $u = e^{-2x}$ $\qquad = -\frac{1}{2}\tan(u) + C$

$du = -2e^{-2x}\, dx$

$= -\frac{1}{2}\tan(e^{-2x}) + C$