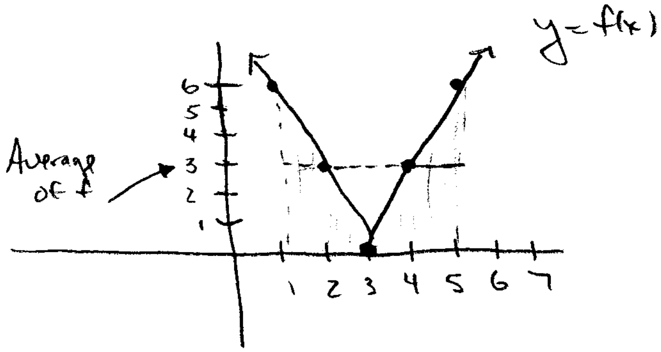
# Fake Test 2A Key



(1) $f(x) = |3x-9|$

$= |3(x-3)|$

$= 3|x-3|$




) $\int_1^5 3|x-3|\,dx = \int_1^3 3(-1)(x-3)\,dx + \int_3^5 3(x-3)\,dx$

$= \left(\frac{-3x^2}{2} + 9x\right)\Big|_1^3 + \left(\frac{3x^2}{2} - 9x\right)\Big|_3^5$

$= \left(-\frac{27}{2} + 27\right) - \left(-\frac{3}{2} + 9\right) + \left(\frac{75}{2} - 45\right) - \left(\frac{27}{2} - 27\right)$

$= -27 + 2(27) + \frac{78}{2} - 54$

$= 27 + 39 - 54$

$= 12$

(b) The average value of $f$ on $[1,5]$ is

$\frac{1}{5-1}\int_1^5 f(x)\,dx = \frac{1}{4}\int_1^5 3|x-3|\,dx = \frac{12}{4} = 3$ from above.

(c) Since the average value of $f$ on $[1,5]$ is 3, the rectangle with height 3 and width 4 has the same area as $\int_1^5 f(x)\,dx$.

2) If $f$ is odd and $\int_1^2 f(x)\,dx = 3$.

Then

$$\int_{-1}^{2} 3f(x) + x\,dx = 3\int_{-1}^{2} f(x)\,dx + \int_{-1}^{2} x\,dx$$

$$= 3\int_{-1}^{1} f(x)\,dx + 3\int_{1}^{2} f(x)\,dx + \int_{-1}^{2} x\,dx$$

$$= 3\cdot 0 + 3\cdot 3 + \left.\frac{x^2}{2}\right|_{-1}^{2}$$

$$= 9 + \left(2 - \tfrac{1}{2}\right)$$

$$= 10\tfrac{1}{2} = \frac{21}{2}$$

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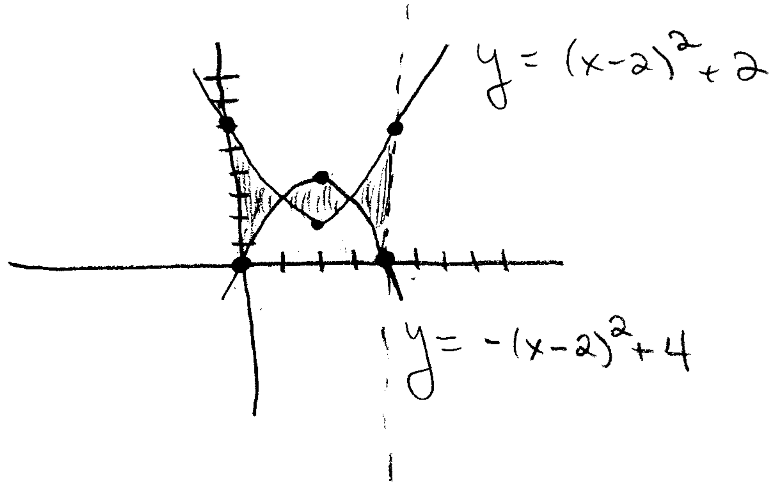
If $g$ is even and $\int_0^3 g(x)\,dx = -2$,

$$\int_{-3}^{3} 2g(x) - 4x^3\,dx = 2\int_{-3}^{3} g(x)\,dx - 4\int_{-3}^{3} x^3\,dx$$

$$= 4\int_{0}^{3} g(x)\,dx - 4\cdot 0$$

$$= -8$$

3 (a)




Where do they meet?

$$(x-2)^2 + 2 = -(x-2)^2 + 4$$

$$2(x-2)^2 = 2$$

$$(x-2) = \pm 1$$

$$x = 1, 3$$

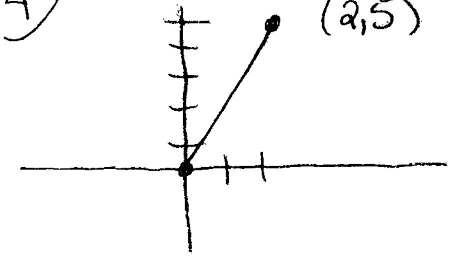
(b)

$$\text{Area} = \int_0^1 \left[(x-2)^2 + 2\right] - \left[-(x-2)^2 + 4\right] dx$$

$$+ \int_1^3 \left[-(x-2)^2 + 4\right] - \left[(x-2)^2 + 2\right] dx$$

$$+ \int_3^4 \left[(x-2)^2 + 2\right] - \left[-(x-2)^2 + 4\right] dx$$

4)




$y = \frac{5}{2}x$

a) $f(x) = \frac{5}{2}x$

$$\text{Volume} = \pi \int_0^2 \left(\tfrac{5}{2}x\right)^2 dx$$

$$= \frac{25}{4\pi} \left.\frac{x^3}{3}\right|_0^2$$

$$= \frac{50}{3\pi}$$

b) $f(y) = \frac{2}{5}y$

$$\text{Volume} = \pi \int_0^5 \left(\tfrac{2}{5}y\right)^2 dy$$

$$= \frac{4\pi}{25} \left.\frac{y^3}{3}\right|_0^5$$

$$= \frac{20\pi}{3}$$

c) Part a) calculates the volume of a cone with base radius 5 and height 2. Part b) calculates the volume of a cone with base radius 2 and height 5. These geometric shapes have different volumes.

(5) $$\int \frac{2x}{\sqrt{3x-1}}\,dx = \int \frac{2\left(\frac{u+1}{3}\right)}{\sqrt{u}}\,\frac{du}{3}$$ (3)

Let $u = 3x-1$ and $x = \frac{u+1}{3}$

$du = 3dx$

$$= \frac{2}{9}\int \frac{u+1}{u^{1/2}}\,du$$

$$= \frac{2}{9}\int u^{1/2} + u^{-1/2}\,du$$

$$= \frac{2}{9}\cdot\frac{2}{3}u^{3/2} + \frac{2}{9}\cdot\frac{2}{1}u^{1/2} + C$$

$$= \frac{4}{27}(3x-1)^{3/2} + \frac{4}{9}(3x-1)^{1/2} + C$$

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(6) $$\int x\left(\cos(2x) - \ln(3x)\right)dx$$

$$= \int x\cos(2x)\,dx - \int x\ln(3x)\,dx$$

$u = x$ $\quad dv = \cos(2x)dx$

$du = dx$ $\quad v = \frac{1}{2}\sin(2x)$

$u = \ln(3x)$ $\quad dv = x\,dx$

$du = \frac{1}{3x}\cdot 3\,dx$ $\quad v = \frac{x^2}{2}$

$$= \frac{x\sin(2x)}{2} - \int \frac{1}{2}\sin(2x)\,dx - \left[\frac{x^2}{2}\ln(3x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx\right]$$

$$= \frac{x\sin(2x)}{2} + \frac{\cos(2x)}{4} - \frac{x^2}{2}\ln(3x) + \frac{x^2}{4} + C$$

⑦ $\int \sec(e^{-2x})\, e^{-2x}\, dx = \int \sec(u)\, \frac{du}{-2}$

Let $u = e^{-2x}$

$du = -2e^{-2x}\, dx$

$= \frac{-1}{2} \ln|\sec(u) + \tan(u)| + C$

$= -\frac{1}{2} \ln|\sec(e^{-2x}) + \tan(e^{-2x})| + C$