Math 16B Key to Fake Test 3a

1 We must 1^{st} divide.

$$
\begin{array}{r|l}
 & -x \quad +5 \\
x^3-4x^2+4x & -x^4+9x^3-27x^2+37x-16 \\
 & \underline{\mp x^4 \pm 4x^3 \mp 4x^2} \\
 & \qquad 5x^3-23x^2+37x \\
 & \qquad \underline{\mp 5x^3 \pm 20x^2 \mp 20x} \\
 & \qquad\qquad -3x^2+17x-16
\end{array}
$$

Thus

$$\int \frac{-x^4+9x^3-27x^2+37x-16}{x^3-4x^2+4x}\,dx = \int -x+5+\frac{-3x^2+17x-16}{x^3-4x^2+4x}\,dx$$

$$= -\frac{x^2}{2}+5x+\int\frac{-3x^2+17x-16}{x^3-4x^2+4x}\,dx.$$

<u>Partial Fractions:</u>

$$\frac{-3x^2+17x-16}{x(x-2)^2} = \frac{A}{x}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$$

i.e. $-3x^2+17x-16 = A(x-2)^2+Bx(x-2)+Cx$

$$= (A+B)x^2+(-4A-2B+C)x+4A$$

Now

$$4A = -16 \Rightarrow A = -4$$

$$A + B = -3 \Rightarrow B = 1$$

$$-4A - 2B + C = 17$$

$$\Rightarrow C = 17 + 4(-4) + 2(1) = 3$$

So that

$$\int \frac{-x^4 + 9x^3 - 27x^2 + 37x - 16}{x^3 - 4x^2 + 4x}\,dx = -\frac{x^2}{2} + 5x + \int \frac{-3x^2 + 17x - 16}{x(x-2)^2}\,dx$$

$$= -\frac{x^2}{2} + 5x - 4\int \frac{1}{x}\,dx + \int \frac{1}{x-2}\,dx$$

$$+ 3\int \frac{1}{(x-2)^2}\,dx$$

$$= -\frac{x^2}{2} + 5x - 4\ln|x| + \ln|x-2| - \frac{3}{x-2} + C$$

(2) (a)

$$\int_3^\infty \frac{1}{\sqrt{2x-3}}\,dx$$

$$= \lim_{b\to\infty} \int_3^b \frac{1}{\sqrt{2x-3}}\,dx$$

$$= \lim_{b\to\infty} \sqrt{2x-3}\,\Big|_3^b = \infty$$

This integral diverges.

(b)

$$\int_{\frac{3}{2}}^5 \frac{1}{\sqrt{2x-3}}\,dx$$

There is a bad spot at $x = 3/2$.

$$= \lim_{a\to\frac{3}{2}^+} \int_a^5 \frac{1}{\sqrt{2x-3}}\,dx$$

$$= \lim_{a\to 3/2^+} \sqrt{2x-3}\,\Big|_a^5$$

$$= \sqrt{7} - 0$$

This integral converges.

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3) (a) S = { HHHH, HHHT, HHTH, HTHH, THHH, TTHH, THTH, THHT, HTTH, HTHT, HHTT, TTTH, TTHT, THTT, HTTT, TTTT }

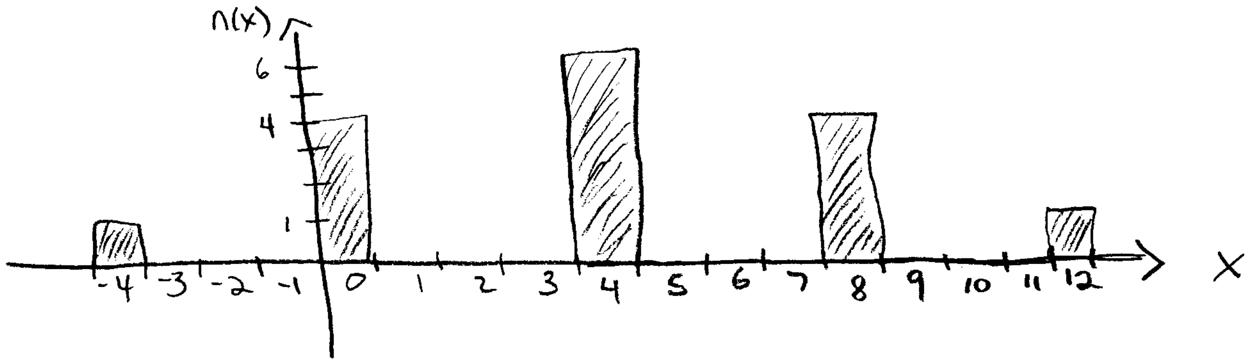
(b)

S	HHHH	HHHT	HHTH	HTHH	THHH	TTHH	THTH	THHT
X	12	8	8	8	8	4	4	4

S	HTTH	HTHT	HHTT	TTTH	TTHT	THTT	HTTT	TTTT
X	4	4	4	0	0	0	0	-4

3) (b)

x	-4	0	4	8	12
n(x)	1	4	6	4	1



(c)

$$\mu = E(x) = -4 \cdot P(-4) + 0 \cdot P(0) + 4 \cdot P(4) + 8 \cdot P(8) + 12 \cdot P(12)$$

$$= -4 \cdot \frac{1}{16} + 0 + 4 \cdot \frac{6}{16} + 8 \cdot \frac{4}{16} + 12 \cdot \frac{1}{16}$$

$$= \frac{64}{16} = 4$$

$$V(x) = (-4-4)^2 \cdot P(-4) + (0-4)^2 \cdot P(0) + (4-4)^2 P(4) + (8-4)^2 \cdot P(8)$$

$$+ (12-4)^2 P(12)$$

$$= 64 \cdot \frac{1}{16} + 16 \cdot \frac{4}{16} + 0 + 16 \cdot \frac{4}{16} + 64 \cdot \frac{1}{16}$$

$$= 16$$

$$\sigma = \sqrt{V(x)} = 4$$

④

(a) $a \int_3^7 (x-3)(7-x)\,dx$

$= a \int_3^7 -x^2+10x-21\,dx$

$= a\left[\frac{-x^3}{3}+5x^2-21x\right]\Big|_3^7$

$= a\,7^2\left(-\frac{7}{3}+5-3\right) - a\,3^2\left(-1+5-7\right)$

$= -a\frac{49}{3} + a\,27 = a\cdot\frac{32}{3}$

For f to be a prob. density, this integral must be 1.

Take $a = \frac{3}{32}$.

(b) $\mu = E(X) = a\int_3^7 (-x^3+10x^2-21x)\,dx$

$= a\left[-\frac{x^4}{4}+\frac{10x^3}{3}-\frac{21x^2}{2}\right]_3^7$

$= a\,7^3\left[-\frac{7}{4}+\frac{10}{3}-\frac{3}{2}\right] - a\,3^3\left[-\frac{3}{4}+\frac{10}{3}-\frac{7}{2}\right]$

$= a\,7^3\cdot\frac{1}{12} + a\,3^3\cdot\frac{11}{12}$

$= a\,\frac{640}{12} = \frac{3}{32}\cdot\frac{640}{12} = \frac{20}{4} = 5$

$$\int_3^m a(x-3)(7-x)\,dx$$

$$= a\left[\frac{-x^3}{3} + 5x^2 - 21x\right]_3^m$$

$$= a\left(\frac{-m^3}{3} + 5m^2 - 21m\right) + 27a$$

$$= \frac{-m^3}{32} + \frac{15}{32}m^2 - \frac{63}{32}m + \frac{81}{32}$$

we want this to be $1/2$.

$$\frac{-m^3}{32} + \frac{15}{32}m^2 - \frac{63m}{32} + \frac{81}{32} = \frac{1}{2}$$

$$\Rightarrow \quad m^3 - 15m^2 + 63m - 65 = 0$$

Check: $m=5$ is a solution.

ⓒ $$V(x) = a\int_3^7 x^2(x-3)(7-x)\,dx - (5)^2$$

$$= a\int_3^7 -x^4 + 10x^3 - 21x^2\,dx \quad -25$$

$$= a\left[\frac{-x^5}{5} + \frac{10x^4}{4} - 7x^3\right]\Big|_3^7 - 25$$

$$= a7^4\left[\frac{-7}{5} + \frac{10}{4} - 1\right] - a3^3\left[\frac{-9}{5} + \frac{30}{4} - 7\right]$$

$$= \frac{a7^4}{10} + \frac{a3^3 \cdot 13}{10} = \frac{3\cdot 7^4 + 3^4\cdot 13}{320}$$

$$\sigma = \sqrt{V(x)} = \sqrt{\frac{3\cdot 7^4 + 3^4\cdot 13}{320}}$$

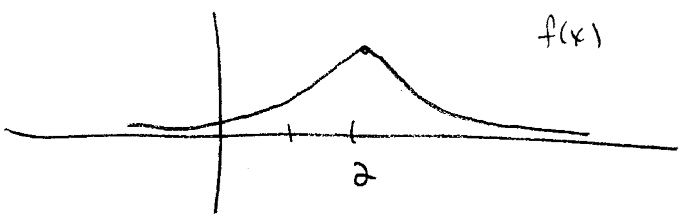
5 $\qquad\qquad\qquad\qquad\qquad\qquad$ 4

(a) $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

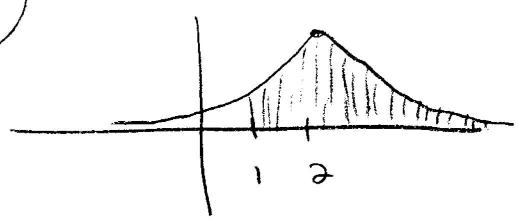
If $\mu = 2$ and $V(x) = 3$, then $\sigma = \sqrt{3}$

i.e.

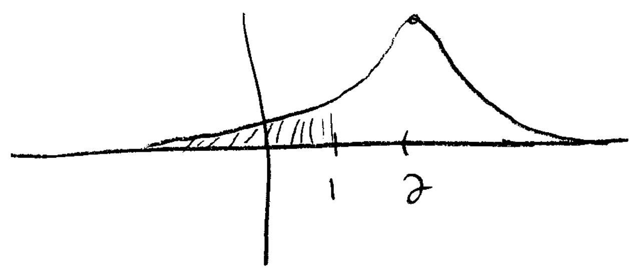
$$f(x) = \frac{1}{\sqrt{6\pi}} e^{\frac{-(x-2)^2}{6}}$$



(b)



$P(x \geq 1)$



$P(x < 1)$

This is bigger because

1 is less than the

mean.