1. Must 1st divide.

$$\begin{array}{r|l} & x + 2 \\ \hline x^2 - x - 6 & x^3 + x^2 - 6x \\ & -x^3 \pm x^2 \pm 6x \\ \hline & 2x^2 \\ & -2x^2 \pm 2x \pm 12 \\ \hline & 2x + 12 \end{array}$$

Thus

$$\int \frac{x(x+3)(x-2)}{(x-3)(x+2)}\,dx = \int x + 2 + \frac{2x+12}{(x-3)(x+2)}\,dx$$

$$= \frac{x^2}{2} + 2x + 2\int \frac{x+6}{(x-3)(x+2)}\,dx$$

Partial Fractions:

$$\frac{x+6}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$x+6 = A(x+2) + B(x-3)$$

$1 = A + B$

$6 = 2A - 3B$

$A = 1 - B$

$6 = 2(1-B) - 3B$

$4 = -5B$

$B = -\frac{4}{5} \Rightarrow A = \frac{9}{5}$

$$= \frac{x^2}{2} + 2x + 2\int \frac{\frac{9}{5}}{x-3}\,dx + 2\int \frac{-\frac{4}{5}}{x+2}\,dx$$

$$= \frac{x^2}{2} + 2x + \frac{18}{5}\ln|x-3| - \frac{8}{5}\ln|x+2| + C$$

(2)

(a) $\int_5^\infty \frac{1}{(x-3)^2}\,dx$

$= \lim_{b\to\infty} \int_5^b \frac{1}{(x-3)^2}\,dx$

$= \lim_{b\to\infty} \left.\frac{-1}{(x-3)}\right|_5^b$

This integral converges.

$= \frac{1}{2}$

(b) $\int_0^5 \frac{1}{(x-3)^2}\,dx$

This integral has a bad spot at $x=3$

$= \int_0^3 \frac{1}{(x-3)^2}\,dx + \int_3^5 \frac{1}{(x-3)^2}\,dx$

$= \lim_{b\to 3^-} \int_0^b (x-3)^{-2}\,dx + \lim_{a\to 3^+} \int_a^5 (x-3)^{-2}\,dx$

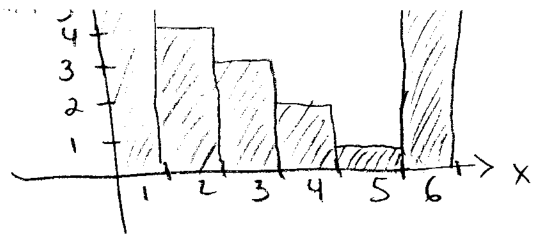
$= \lim_{b\to 3^-} \left.\frac{-1}{x-3}\right|_0^b + \lim_{a\to 3^+} \left.\frac{-1}{x-3}\right|_a^5$

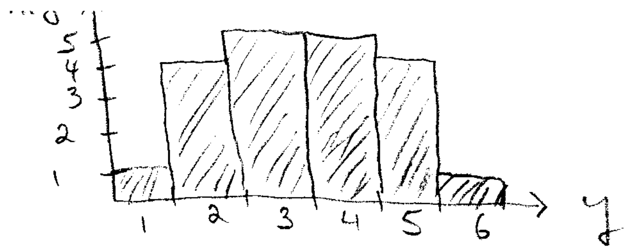
$= \infty + \infty$

Both of those integrals diverge so the entire integral diverges.

3

a





2

b

x	1	2	3	4	5	6
$P(x)$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{4}$

y	1	2	3	4	5	6
$P(y)$	$\frac{1}{20}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{20}$

c

$$\mu = E(x) = 1\cdot\frac{1}{4} + 2\cdot\frac{1}{5} + 3\cdot\frac{3}{20} + 4\cdot\frac{1}{10} + 5\cdot\frac{1}{20} + 6\cdot\frac{1}{4} = \frac{65}{20} = 3\tfrac{1}{4}$$

$$\mu = E(y) = 1\cdot\frac{1}{20} + 2\cdot\frac{1}{5} + 3\cdot\frac{1}{4} + 4\cdot\frac{1}{4} + 5\cdot\frac{1}{5} + 6\cdot\frac{1}{20} = \frac{70}{20} = 3\tfrac{1}{2}$$

$$V(x) = \left(1-\frac{13}{4}\right)^2\cdot\frac{1}{4} + \left(2-\frac{13}{4}\right)^2\cdot\frac{1}{5} + \left(3-\frac{13}{4}\right)^2\cdot\frac{3}{20} + \left(4-\frac{13}{4}\right)^2\cdot\frac{1}{10} + \left(5-\frac{13}{4}\right)^2\cdot\frac{1}{20} + \left(6-\frac{13}{4}\right)^2\cdot\frac{1}{4}$$

$$V(y) = \left(1-\frac{7}{2}\right)^2\cdot\frac{1}{20} + \left(2-\frac{7}{2}\right)^2\cdot\frac{1}{5} + \left(3-\frac{7}{2}\right)^2\cdot\frac{1}{4} + \left(4-\frac{7}{2}\right)^2\cdot\frac{1}{4} + \left(5-\frac{7}{2}\right)^2\cdot\frac{1}{5} + \left(6-\frac{7}{2}\right)^2\cdot\frac{1}{20}$$

(4) $f(x) = e^{-x/5}$

(a) $$\int_0^b e^{-x/5}\,dx = -5e^{-x/5}\Big|_0^b$$

$$= -5e^{-b/5} + 5$$

If we want f to be a prob. density, this integral must be 1

$$1 = -5e^{-b/5} + 5$$

$$5e^{-b/5} = 4$$

$$\frac{-b}{5} = \ln(4/5)$$

$$b = -5\ln(4/5)$$

(b) $$\mu = E(x) = \int_0^b x e^{-x/5}\,dx = -5xe^{-x/5}\Big|_0^b + \int_0^b 5e^{-x/5}\,dx$$

$u = x \quad dv = e^{-x/5}dx$

$du = dx \quad v = -5e^{-x/5}$

$$= -5be^{-b/5} - 25e^{-x/5}\Big|_0^b$$

$$= 25 - 5be^{-b/5} - 25e^{-b/5}$$

$$= 25 - 4b - 20$$

$$= 5 - 4b$$

$$= 5 + 20\ln(4/5).$$

<u>Note</u> $e^{-b/5} = e^{\frac{5\ln(4/5)}{5}} = 4/5$

(b) The median, m, satisfies:

$$\int_0^m e^{-x/5}\,dx = 1/2$$

we already calculated that $\int_0^m e^{-x/5}\,dx = 5 - 5e^{-m/5}$

so

$$5 - 5e^{-m/5} = 1/2$$

$$5e^{-m/5} = \frac{9}{2}$$

$$-m/5 = \ln\left(\tfrac{9}{10}\right)$$

$$m = -5\ln\left(\tfrac{9}{10}\right)$$

(c) $$V(x) = \int_0^b x^2 e^{-x/5}\,dx - \mu^2$$

Let $u = x^2$ $\quad dv = e^{-x/5}\,dx$

$du = 2x\,dx \quad v = -5e^{-x/5}$

$$V(x) = -5x^2 e^{-x/5}\Big|_0^b + \int_0^b 10x\,e^{-x/5}\,dx - \mu^2$$

$$= -5b^2 e^{-b/5} + 10\int_0^b x e^{-x/5}\,dx - \mu^2$$

$\boxed{\text{from before}}$

$$= -4b^2 + 10\mu - \mu^2 = -4(-5\ln(4/5))^2 + 10(5+20\ln(4/5))$$
$$- (5+20\ln(4/5))^2$$

$\sigma = \sqrt{V(x)}$.

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5) If x is a continuous random variable which is uniformly distributed over $[a,b]$, then the probability density function f for x is

$$f(x) = \frac{1}{b-a}$$

<u>hen</u>

$$\mu = E(x) = \int_a^b x \cdot \frac{1}{b-a}\,dx = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$$

$$\int_a^m \frac{1}{b-a}\,dx = 1/2 \quad \underline{\text{means}} \quad \frac{m-a}{b-a} = 1/2$$

$$m = a + \frac{b-a}{2} = \frac{a+b}{2}$$

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and

$$V(x) = \int_a^b x^2 \cdot \frac{1}{b-a}\,dx - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^3-a^3}{3(b-a)} - \frac{(a+b)^2}{4} = \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}} - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{4(b^2+ab+a^2)}{12} - \frac{3(a^2+2ab+b^2)}{12}$$

$$= \frac{(b-a)^2}{12}$$