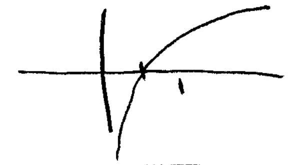
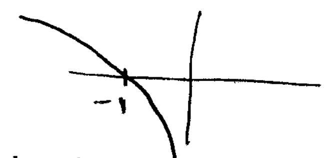
KEY

The Basic graph here is $y = \ln(x)$



One can reflect this: $y = \ln(-x)$



MATH 16B:
TEST 1

FALL 2006

(1) Graph the following function:

$$f(x) = \ln(3 - x) + 5.$$

To receive full credit, you must plot the x-intercept, the y-intercept, one additional point, and indicate all vertical and (or) horizontal asymptotes.

Calculate intercepts:

If $x=0$, then $f(0) = \ln(3) + 5$

$\Rightarrow (0, \ln(3)+5)$ is the y-intercept.

For what x value is

$0 = \ln(3-x)+5$

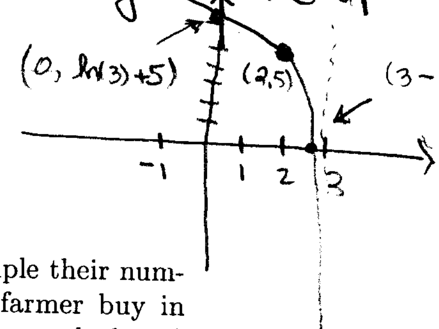
$\Rightarrow \ln(3-x) = -5$

$3 - x = e^{-5}$

$x = 3 - e^{-5}$

$\Rightarrow (3-e^{-5}, 0)$ is the x-intercept

We are interested in the reflected graph shifted 3 right and 5 up



Horizontal Asymptote $x=3$

(2) A certain breed of bunny rabbits is known to quadruple their numbers within a month. How many bunnies must a farmer buy in January so that he has 2,500 bunnies for Easter three months later? (Assume the bunnies reproduce according to an exponential growth model.) To receive full credit, you must write out the exponential growth model and label all variables.

The growth model is

$y(t) = Ce^{kt}$ where t is the number of months and $y(t)$ is the number of bunnies t months later. Clearly, the initial number of bunnies is always $y(0) = Ce^{k\cdot 0} = C$.

If they quadruple in 1 month, then one can calculate the rate k:

$4C = y(1) = Ce^{k} \Rightarrow 4 = e^{k} \Rightarrow k = \ln(4)$.

We must find C. We know $2500 = y(3) = Ce^{\ln(4)\cdot 3}$

$\Rightarrow C = 2500/e^{3\cdot\ln(4)} = \dfrac{2500}{4^3} \approx 39.0625$

$y(t) = \dfrac{2500}{64} e^{\ln(4)t}$

(3) Find the derivative of

$$f(x) = (x^2 - \sqrt[5]{x} + 6)^{\tan(2x)}.$$

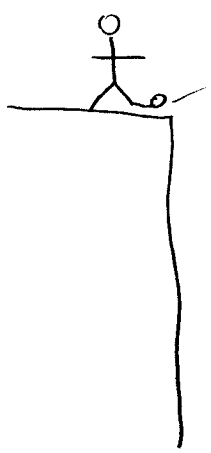
To differentiate this function, we rewrite it as an exponential:

$$f(x) = (x^2 - \sqrt[5]{x} + 6)^{\tan(2x)} = e^{\ln\left[(x^2 - \sqrt[5]{x} + 6)^{\tan(2x)}\right]} = e^{\tan(2x)\ln[x^2 - \sqrt[5]{x} + 6]}$$

$$\Rightarrow \frac{df}{dx}(x) = e^{\tan(2x)\ln[x^2 - \sqrt[5]{x} + 6]} \cdot \frac{d}{dx}\left(\tan(2x)\ln[x^2 - \sqrt[5]{x} + 6]\right)$$

$$= (x^2 - \sqrt[5]{x} + 6)^{\tan(2x)}\left[2\sec^2(2x)\ln[x^2 - \sqrt[5]{x} + 6] + \frac{\tan(2x)}{x^2 - \sqrt[5]{x} + 6}\left(2x - \frac{1}{5x^{4/5}}\right)\right]$$

(4) A woman kicks a ball off the top of a building. She kicks it with an initial velocity of 48 feet per second. If the ball hits the ground 5 seconds later, how tall is the building? To receive full credit, you must also determine the position function, $s(t)$, describing the height of this ball t seconds after it has been kicked.



We want to find the position function $s(t)$.

We know

$$s''(t) = -32 \quad \text{(only gravity acts on the ball)}$$

$$\Rightarrow \quad s'(t) = -32t + C \quad \text{(integrate to find velocity)}$$

$$C = \underset{\substack{\uparrow \\ \text{initial velocity}}}{s'(0)} = 48$$

$$\Rightarrow \quad s'(t) = -32t + 48$$

$$\Rightarrow \quad s(t) = -16t^2 + 48t + C \quad \text{(integrate to find position)}$$

$$C = \underset{\substack{\uparrow \\ \text{initial position}}}{s(0)} = \text{Height of building.}$$

We know $0 = s(5) = -16(5)^2 + 48(5) + C \Rightarrow C = 160$

and the position function is $s(t) = -16t^2 + 48t + 160$

(5) Integrate the following:

$$\int \frac{-21+24x-9x^2}{\sqrt[3]{x^3-4x^2+7x-5}}dx.$$

Use substitution: Let $u = x^3-4x^2+7x-5 \Rightarrow du = (3x^2-8x+7)dx$
$\Rightarrow -3\,du = (-9x^2+24x-21)\,dx$

$$\Rightarrow \int \frac{-21+24x-9x^2}{\sqrt[3]{x^3-4x^2+7x-5}}dx = \int \frac{1}{u^{1/3}}(-3)du = -3\int u^{-1/3}du$$
$$= -3\frac{u^{2/3}}{2/3} + C$$
$$= -\frac{9}{2}(x^3-4x^2+7x-5)^{2/3} + C$$

(6) Integrate the following:

$$\int \frac{\sin(3\sqrt{\theta}) + e^{-\sqrt{\theta}}}{\sqrt{\theta}}d\theta.$$

Break into two integrals and use two substitutions:

$$\int \frac{\sin(3\sqrt{\theta}) + e^{-\sqrt{\theta}}}{\sqrt{\theta}}d\theta = \int \frac{\sin(3\sqrt{\theta})}{\sqrt{\theta}}d\theta + \int \frac{e^{-\sqrt{\theta}}}{\sqrt{\theta}}d\theta = \frac{2}{3}\int \sin(u)du - 2\int e^{w}dw$$

Let $u = 3\sqrt{\theta}$
$\Rightarrow du = \frac{3}{2\sqrt{\theta}}$

Let $w = -\sqrt{\theta}$
$dw = \frac{-1}{2\sqrt{\theta}}d\theta$

$$= -\frac{2}{3}\cos(u) - 2e^{w} + C$$
$$= -\frac{2}{3}\cos(3\sqrt{\theta}) - 2e^{-\sqrt{\theta}} + C$$

(7) Integrate the following:

$$\int \frac{\sin(2x)}{\cos(2x)}dx.$$

Let $u = \cos(2x)$
$du = -2\sin(2x)dx$

Then

$$\int \frac{\sin(2x)}{\cos(2x)}dx = -\frac{1}{2}\int \frac{1}{u}du = -\frac{1}{2}\ln|u| + C$$
$$= -\frac{1}{2}\ln|\cos(2x)| + C$$