

**MATH 16B:  
FAKE TEST 3A**

SPRING 2007

(1) Integrate the following:

$$\int \frac{-x^4 + 9x^3 - 27x^2 + 37x - 16}{x^3 - 4x^2 + 4x} dx.$$

(2) Determine whether the following integrals converge or diverge. If they converge, find their value.

a)  $\int_3^{\infty} \frac{1}{\sqrt{2x-3}} dx,$

and

b)  $\int_{3/2}^5 \frac{1}{\sqrt{2x-3}} dx.$

(3) Consider an experiment where a coin is flipped 4 times.

- a) Write out the sample space of this experiment.
- b) Define a random variable  $x$  which assigns 3 points to each head flipped and -1 point for each tail. Write a frequency table for  $x$  and plot its frequency distribution.
- c) Find the expected value of  $x$  (i.e., the mean), the variance, and the standard deviation.

(4) Consider the function

$$f(x) = a(x - 3)(7 - x).$$

- a) For what value of  $a$  is  $f$  a probability density function on  $[3, 7]$ ?
- b) For this value of  $a$ , find the mean and median of the continuous random variable  $x$  whose density is  $f$ .
- c) For this value of  $a$ , find the variance and standard deviation of  $x$ .

(5) Suppose that  $x$  is a continuous random variable with normal distribution.

- a) If  $x$  has mean 2 and variance 3, write the probability density function  $f$  for  $x$  and sketch its graph.
- b) Graph  $P(x \geq 1)$  and  $P(x < 1)$  by shading the appropriate regions. Which is bigger? Why?