MATH 16B:
GRAPHING HANDOUT

SPRING 2007

Understanding the graphs of certain basic functions is essential in Calculus. Once the basic graphs are understood, more complicated looking *translations* and *reflections* are also easily understood.

Translations: Suppose f is a function whose graph is known and $c > 0$ is a constant.

(1) Vertical Shifts:
i) $y = f(x) + c$ shifts the graph of $y = f(x)$ a distance c units upwards.
ii) $y = f(x) - c$ shifts the graph of $y = f(x)$ a distance c units downwards.
Note: Vertical translations do not effect vertical asymptotes, but they will move horizontal asymptotes!

(2) Horizontal Shifts:
i) $y = f(x - c)$ shifts the graph of $y = f(x)$ a distance c units to the right.
ii) $y = f(x + c)$ shifts the graph of $y = f(x)$ a distance c units to the left.
Note: Horizontal translations do not effect horizontal asymptotes, but they will move vertical asymptotes!

Reflections: Suppose f is a function whose graph is known.
i) $y = -f(x)$ reflects the graph of $y = f(x)$ about the x-axis. This kind of reflection will not effect vertical asymptotes, but it may effect horizontal asymptotes.
ii) $y = f(-x)$ reflects the graph of $y = f(x)$ about the y-axis. This kind of reflection will not effect horizontal asymptotes, but it may effect vertical asymptotes.

Exercise 1. *Let the function f be given by $f(x) = x^2$.*

Graph the following translations:

i) $y = f(x) + 3 = x^2 + 3$,
ii) $y = f(x) - 4 = x^2 - 4$,
iii) $y = f(x - 2) = (x - 2)^2$,
iv) $y = f(x + 1) = (x + 1)^2$.

Graph the following reflections:

i) $y = -f(x) = -x^2$,
ii) $y = f(-x) = (-x)^2$,

Graph the following combinations:

i) $y = -f(x + 3) + 2 = -(x + 3)^2 + 2$,
ii) $y = f(-x + 3) - 1 = (-x + 3)^2 - 1$.

Exercise 2. *Let the function g be given by $g(x) = 3^x$.*

Graph the following translations:

i) $y = g(x) + 3 = 3^x + 3$,
ii) $y = g(x) - 4 = 3^x - 4$,
iii) $y = g(x - 2) = 3^{x-2}$,
iv) $y = g(x + 1) = 3^{x+1}$.

Graph the following reflections:

i) $y = -g(x) = -3^x$,
ii) $y = g(-x) = 3^{-x}$,

Graph the following combinations:

i) $y = -g(x + 3) + 2 = -3^{x+3} + 2$,
ii) $y = g(-x + 3) - 1 = 3^{-x+3} - 1$.