Homework 1 key (the rest)

Section 4.4:

#52 $e^{\ln(x^2)} - 9 = 0$

$\Rightarrow x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

#57 $300\, e^{-0.2t} = 700$

$e^{-0.2t} = \frac{7}{3}$

$-0.2t = \ln\left(\frac{7}{3}\right)$

$t = -\frac{1}{0.2}\ln\left(\frac{7}{3}\right)$

#62 $\dfrac{50}{1 + 12\, e^{-0.02x}} = 10.5$

$\Rightarrow \quad 50 = 10.5\left(1 + 12\, e^{-0.02x}\right)$

$\Rightarrow \quad \frac{50}{10.5} = 1 + 12\, e^{-0.02x}$

$\Rightarrow \quad \frac{50}{10.5} - 1 = 12\, e^{-0.02x}$

$$\Rightarrow \quad e^{-0.02x} = \frac{1}{12}\left(\frac{50}{10.5} - 1\right)$$

$$\Rightarrow \quad -0.02x = \ln\left[\frac{1}{12}\left(\frac{50}{10.5} - 1\right)\right]$$

$$\Rightarrow \quad x = \frac{-1}{0.02}\ln\left[\frac{1}{12}\left(\frac{50}{10.5} - 1\right)\right]$$

#68 $\quad 2000\left(1 + \frac{0.06}{12}\right)^{12t} = 10,000$

$$\Rightarrow \quad \left(1 + \frac{0.06}{12}\right)^{12t} = \frac{10,000}{2,000} = 5$$

$$\Rightarrow \quad 12t \ln\left[1 + \frac{0.06}{12}\right] = \ln[5]$$

$$\Rightarrow \quad t = \frac{1}{12}\frac{\ln[5]}{\ln\left[1 + \frac{0.06}{12}\right]}$$

Sec. 4.5

#2 $\quad y = \ln\left(x^{5/2}\right) = \frac{5}{2}\ln(x)$

$$\Rightarrow \quad y' = \frac{5}{2x} \quad \Rightarrow y'(1) = \frac{5}{2}.$$

#9 $\quad y = \ln\left(\sqrt{x^4 - 4x}\right) = \frac{1}{2}\ln(x^4 - 4x)$

$$y' = \frac{1}{2} \cdot \frac{1}{x^4 - 4x} \cdot (4x^3 - 4)$$

#12 $\quad y = \left(\ln(x^2)\right)^2$

$$\Rightarrow y' = 2\left(\ln(x^2)\right) \cdot \frac{d}{dx}\left(\ln(x^2)\right)$$

$$= 2\left(\ln(x^2)\right) \cdot \frac{1}{x^2} \cdot 2x$$

$$= \frac{4\ln(x^2)}{x}$$

Section 4.5 (cont.)

#18 $y = \ln\left(\frac{x^2}{x^2+1}\right) = \ln(x^2) - \ln(x^2+1)$

$\Rightarrow y' = \frac{1}{x^2} \cdot 2x - \frac{1}{x^2+1} \cdot 2x$

$\Rightarrow y' = \frac{2}{x} - \frac{2x}{x^2+1}$

#22 $y = \ln\left(x\sqrt{4+x^2}\right) = \ln(x) + \ln\left(\sqrt{4+x^2}\right)$

$\Rightarrow y = \ln(x) + \frac{1}{2}\ln(4+x^2)$

$\Rightarrow y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{4+x^2} \cdot 2x$

$\Rightarrow y' = \frac{1}{x} + \frac{x}{4+x^2}$

#40 $g(x) = \log_5(x) = \frac{\ln(x)}{\ln(5)}$

$\Rightarrow g'(x) = \frac{1}{\ln(5)} \cdot \frac{1}{x}$

#44 $f(x) = 10^{x^2}$

$f'(x) = \ln(10) \cdot 10^{x^2} \cdot \frac{d}{dx}(x^2)$

$f'(x) = 2x\ln(10) \cdot 10^{x^2}$

#48 $y = \frac{\ln(x)}{x} \Rightarrow y' = \frac{x \cdot \frac{1}{x} - \ln(x)}{x^2} = \frac{1 - \ln(x)}{x^2}$

$y'(e) = \frac{1 - \ln(e)}{e^2} = 0$

$m = 0 \quad P = (e, 1/e) \Rightarrow y - y_1 = m(x - x_1)$

$\boxed{y = 1/e}$

#54 $4xy + \ln(x^2 y) = 7$

Take $\frac{d}{dx}$ of both sides $\Rightarrow 4y + 4x\frac{dy}{dx} + \frac{1}{x^2 y} \cdot \left(2xy + x^2 \frac{dy}{dx}\right) = 0$

$\Rightarrow 4y + \frac{2}{x} + \left(4x + \frac{1}{y}\right)\frac{dy}{dx} = 0$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-4y - \frac{2}{x}}{4x + \frac{1}{y}}}$

#66 $f(x) = x^2 \log_3(x) = \frac{x^2 \ln(x)}{\ln(3)}$

$f'(x) = 2x \frac{\ln(x)}{\ln(3)} + \frac{x^2 \cdot \frac{1}{x}}{\ln(3)} = \frac{2x \ln(x)}{\ln(3)} + \frac{x}{\ln(3)}$

$f'(1) = \frac{2(1)\ln(1)}{\ln(3)} + \frac{1}{\ln(3)} = \frac{1}{\ln(3)} = m$

$y - 0 = m(x - 1)$

$\boxed{y = \frac{1}{\ln(3)}(x - 1)}$