Key to HW3

Section 5.3:

(4)

$$\int e^{-0.25x}\,dx = \int e^{u}\frac{du}{-0.25} = \frac{-1}{0.25}e^{-0.25x} + C$$

Note: $0.25 = \frac{1}{4}$

Let $u = -0.25x$

$du = -0.25\,dx$

$$= -4e^{\frac{-x}{4}}$$

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(8)

$$\int (2x+1)\,e^{x^2+x}\,dx = \int e^{u}\,du = e^{x^2+x} + C$$

Let $u = x^2 + x$

$du = (2x+1)\,dx$

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(12)

$$\int 3e^{-(x+1)}\,dx = \int 3e^{u}\frac{du}{-1} = -3e^{-(x+1)} + C$$

Let $u = -(x+1)$

$du = -dx$

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(14)

$$\int \frac{1}{x-5}\,dx = \int \frac{1}{u}\,du = \ln|x-5| + C$$

Let $u = x-5$

$du = dx$

(16) $\int \frac{1}{6x-5}\,dx = \int \frac{1}{u}\,\frac{du}{6} = \frac{1}{6}\ln|6x-5| + C$

Let $u = 6x-5$

$du = 6\,dx$

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(18) $\int \frac{x^2}{3-x^3}\,dx = \int \frac{1}{u}\,\frac{du}{-3} = -\frac{1}{3}\ln|3-x^3| + C$

Let $u = 3-x^3$

$du = -3x^2\,dx$

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(20) $\int \frac{x}{x^2+4}\,dx = \int \frac{1}{u}\,\frac{du}{2} = \frac{1}{2}\ln|x^2+4| + C$

Let $u = x^2+4$

$du = 2x\,dx$

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(22) $\int \frac{x^2+2x+3}{x^3+3x^2+9x+1}\,dx = \int \frac{1}{u}\,\frac{du}{3} = \frac{1}{3}\ln|x^3+3x^2+9x+1| + C$

Let $u = x^3+3x^2+9x+1$

$du = (3x^2+6x+9)\,dx$

$du = 3(x^2+2x+3)\,dx$

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(24) $\int \frac{e^x}{1+e^x}\,dx = \int \frac{1}{u}\,du = \ln|1+e^x| + C$

Let $u = 1+e^x$

$du = e^x\,dx$

Section 8.5

4 $\int \theta^2 + \sec^2\theta \, d\theta = \frac{\theta^3}{3} + \tan(\theta) + C$

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8 $\int \cos(6x)\,dx = \int \cos(u)\frac{du}{6} = \frac{1}{6}\sin(6x) + C$

Let $u = 6x$

$du = 6dx$

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10 $\int x\sin(x^2)\,dx = \int \sin(u)\frac{du}{2} = -\frac{1}{2}\cos(x^2) + C$

Let $u = x^2$

$du = 2x\,dx$

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16 $\int \sqrt{\cot(x)}\cdot\csc^2(x)\,dx = \int u^{1/2}\frac{du}{-1} = -\frac{2}{3}(\cot(x))^{3/2} + C$

Let $u = \cot(x)$

$du = -\csc^2(x)\,dx$

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22 $\int \frac{\sin(x)}{\cos^2(x)}\,dx = \int \frac{1}{u^2}\frac{du}{-1} = \frac{1}{\cos(x)} + C$

$= \sec(x) + C$

Let $u = \cos(x)$

$du = -\sin(x)\,dx$

26 $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \int \sin(u)\cdot 2\,du = -2\cos(\sqrt{x}) + C$

Let $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$

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34 $\displaystyle\int (\csc(2\theta) - \cot(2\theta))^2\,d\theta = \int \csc^2(2\theta) - 2\csc(2\theta)\cot(2\theta) + \cot^2(2\theta)\,d\theta$

Multiply out

$\displaystyle = \int \csc^2(2\theta)\,d\theta - 2\int \csc(2\theta)\cot(2\theta)\,d\theta + \int \cot^2(2\theta)\,d\theta$

The 1st two integrals are "easy".

Let $u = 2\theta$
Then $du = 2d\theta$

$\displaystyle = \int \csc^2(u)\frac{du}{2} - 2\int \csc(u)\cot(u)\frac{du}{2} + \int \cot^2(u)\frac{du}{2}$

$\displaystyle = -\frac{1}{2}\cot(u) - 2\cdot\left(\frac{1}{2}\right)\cdot(-\csc(u)) + \frac{1}{2}\int \cot^2(u)\,du$

use Trig. identity
$\sin^2(x) + \cos^2(x) = 1$
implies
$1 + \cot^2(x) = \csc^2(x)$
or $\boxed{\cot^2(x) = \csc^2(x) - 1}$

$\displaystyle = -\frac{1}{2}\cot(u) + \csc(u) + \frac{1}{2}\int \csc^2(u) - 1\,du$

$\displaystyle = -\frac{1}{2}\cot(u) + \csc(u) - \frac{\cot(u)}{2} - \frac{u}{2} + C$

sub. $u = 2\theta$ $\quad = \boxed{-\cot(2\theta) + \csc(2\theta) - \theta + C}$