(1) Differentiate

$$y = \frac{2+\ln(x)}{x}$$

$$y' = \frac{x(2+\ln(x))' - (2+\ln(x))(x)'}{x^2} = \frac{1 - 2 - \ln(x)}{x^2}$$

$$= \frac{-1-\ln(x)}{x^2}$$

Note:

(i) $y = \frac{2+\ln(x)}{x} \Rightarrow xy = 2+\ln(x)$

and

(ii) $y' = \frac{-1-\ln(x)}{x^2} \Rightarrow x^2y' = -1-\ln(x)$

Now check

$$x^2y' + xy = -1 - \ln(x) + 2 + \ln(x) = 1 \quad \checkmark$$

(2) This is separable

$$y' = \frac{x^2 e^{2y}}{1+x^3}$$

$$\Rightarrow \quad e^{-2y} y' = \frac{x^2}{1+x^3}$$

Let $u = 1+x^3$
$du = 3x^2 dx$

i.e.

$$\int e^{-2y}\, dy = \int \frac{x^2}{1+x^3}\, dx$$

$$\boxed{-\frac{1}{2} e^{-2y} = \frac{1}{3} \ln|1+x^3| + C}$$

This is the general solution

The particular solution which passes through (0,0) satisfies.

$$-\frac{1}{2} e^{-2(0)} = \frac{1}{3}\ln(1+0) + C$$

i.e.

$$C = -1/2$$

Thus

$$\boxed{-\frac{1}{2} e^{-2y} = \frac{1}{3} \ln|1+x^3| - 1/2}$$

(3) This is 1st order linear.

$$y' + \frac{2}{x}y = \frac{\cos(x)}{x^2} \qquad \text{for } x>0$$

$$u = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = e^{\ln(x^2)} = x^2$$

Thus

$$y = \frac{1}{u}\int \frac{\cos(x)}{x^2}\cdot u(x)\,dx$$

$$= \frac{1}{x^2}\int \cos(x)\,dx$$

$$= \frac{1}{x^2}\left[\sin(x) + C\right]$$

④

$\frac{dN}{dt} = K N(500-N)$ This is separable.

$\frac{1}{N(500-N)} \frac{dN}{dt} = K$

$\int \frac{1}{N(500-N)} dN = Kt + C$

Note:

$\frac{1}{N(500-N)} = \frac{A}{N} + \frac{B}{500-N}$

$1 = (500-N)A + BN$

$A = \frac{1}{500} \Rightarrow B = \frac{1}{500}$

$\Rightarrow \frac{1}{500} \ln|N| - \frac{1}{500} \ln|500-N| = Kt + C$

or

$\frac{1}{500} \ln\left|\frac{N}{500-N}\right| = Kt + C$

$\ln\left|\frac{N}{500-N}\right| = 500Kt + C$

$\left|\frac{N}{500-N}\right| = e^{500Kt + C}$

$\frac{N}{500-N} = \pm e^{C} \cdot e^{500Kt}$ → Let this be A.

$N = Ae^{500Kt}(500-N)$

or

$$N(1+Ae^{500kt}) = 500Ae^{500kt}$$

and so

$$N = \frac{500Ae^{500kt}}{1+Ae^{500kt}} = \frac{500}{1+Be^{-500kt}} \quad \text{where } B = \frac{1}{A}$$

Now If $N(0)=100$, then

$$100 = \frac{500}{1+B} \quad \text{i.e.} \quad 1+B = 5$$

$$B = 4$$

and

$N(4)=200$

means

$$200 = \frac{500}{1+4e^{-500k(4)}}$$

$$1+4e^{-2000k} = \frac{5}{2}$$

$$e^{-2000k} = \frac{3}{8}$$

$$-2000k = \ln\left(\frac{3}{8}\right)$$

$$k = \frac{-1}{2000}\ln\left(\frac{3}{8}\right)$$

So N is as above with $B=4$ and $k = -\frac{1}{2000}\ln(3/8)$

(5) Consider

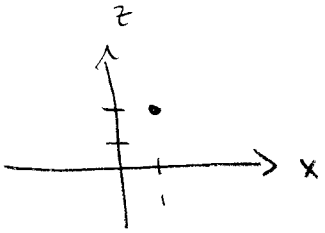
$$(x-1)^2 + (y-3)^2 + (z-2)^2 = 25$$

he trace for

$y = -2$

$$(x-1)^2 + (-5)^2 + (z-2)^2 = 25$$

$$(x-1)^2 + (z-2)^2 = 0$$ is a point.

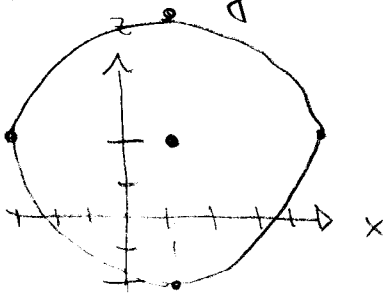


Trace for $y = 0$

$$(x-1)^2 + (-3)^2 + (z-2)^2 = 25$$

$$(x-1)^2 + (z-2)^2 = 16$$

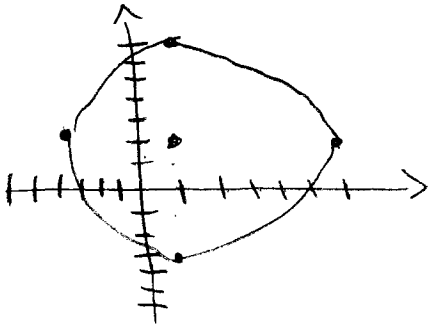
Circle of radius 4 with center (1,2)



Trace for $y = 3$

$$(x-1)^2 + 0 + (z-2)^2 = 25$$

Circle of radius 5 with center (1,2)



Trace for $y = 6$

$$(x-1)^2 + (3)^2 + (z-2)^2 = 25$$

Same as $y = 0$

Trace for $y = 8$

$$(x-1)^2 + (5)^2 + (z-2)^2 = 25$$

Same as $y = -2$