Key to Fake Test 2A

1) $f(x,y) = \sqrt{36-9x^2-4y^2}$

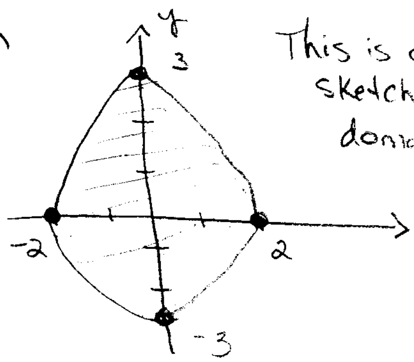
This function is defined when $0 \le 36-9x^2-4y^2$

$$9x^2+4y^2 \le 36$$

$$\boxed{\frac{x^2}{4}+\frac{y^2}{9} \le 1}$$

The set of points for which this equation is satisfied is the domain of f.

a)



This is a sketch of the domain of f.

b) $f(x,y) = c \Rightarrow c = \sqrt{36-9x^2-4y^2}$

$$\Rightarrow 9x^2+4y^2 = 36-c^2$$

$$\Rightarrow \frac{x^2}{\frac{36-c^2}{9}} + \frac{y^2}{\frac{36-c^2}{4}} = 1$$

For each c, the level curve is an ellipse centered at the origin.

if $c=0$, then

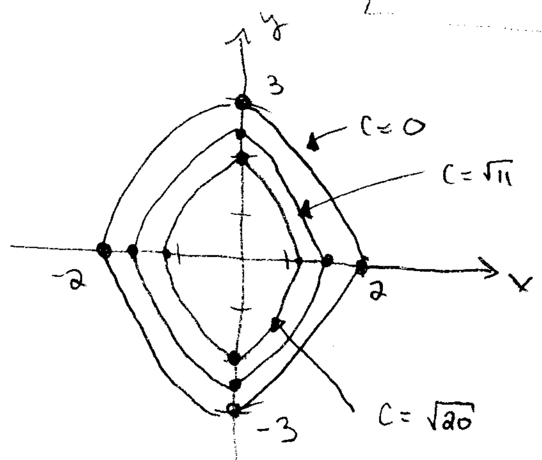
$$\frac{x^2}{4}+\frac{x^2}{9}=1$$

if $c=\sqrt{11}$, then

$$\boxed{\frac{x^2}{\frac{25}{9}} + \frac{y^2}{\frac{25}{4}} = 1}$$

if $c=\sqrt{20}$, then

$$\boxed{\frac{x^2}{\frac{16}{9}} + \frac{y^2}{4} = 1}$$



2)

$$f(x,y,z) = \frac{\ln(x^3 - y^2 + z)}{xyz}$$

$$f_x(x,y,z) = \frac{xyz\left(\frac{1}{x^3-y^2+z} \cdot 3x^2\right) - \ln(x^3-y^2+z)(yz)}{x^2y^2z^2}$$

$$f_y(x,y,z) = \frac{xyz\left(\frac{1}{x^3-y^2+z} \cdot (-2y)\right) - \ln(x^3-y^2+z)(xz)}{x^2y^2z^2}$$

$$f_z(x,y,z) = \frac{xyz\left(\frac{1}{x^3-y^2+z} (1)\right) - \ln(x^3-y^2+z)(xy)}{x^2y^2z^2}$$

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3) $f(x,y) = x^2 + y^2 + x^2y + 4$

$$\left.\begin{aligned} f_x(x,y) &= 2x + 2xy = 2x(1+y) \\ f_y(x,y) &= 2y + x^2 \end{aligned}\right\}$$

Both partials always exist, so the only critical points arise from $f_x = f_y = 0$.

$\boxed{f_x = 0} \Rightarrow 2x(1+y) = 0 \Rightarrow$ either $x = 0$ <u>or</u> $y = -1$.

$\boxed{f_y = 0} \Rightarrow 2y + x^2 = 0$ if $x = 0 \Rightarrow y = 0$.

if $y = -1 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$.

3 (cont.)

So there are 3 critical points:

$P_1 = (0,0)$, $P_2 = (\sqrt{2}, -1)$, and $P_3 = (-\sqrt{2}, -1)$

Note:

$$f_{xx}(x,y) = 2(1+y)$$
$$f_{xy}(x,y) = 2x$$
$$f_{yy}(x,y) = 2$$

So

$$d(x,y) = f_{xx}(x,y) \cdot f_{yy}(x,y) - (f_{xy}(x,y))^2$$
$$= 2x \cdot 2(1+y) - 4x^2$$
$$= 4x[1+y-x]$$

For P_1

$d(0,0) = 0 \Rightarrow$ we know nothing

For P_2

$d(\sqrt{2}, -1) = 4\sqrt{2}(1-1-\sqrt{2}) = -8 < 0 \Rightarrow P_2$ is a saddle point

For P_3

$d(-\sqrt{2}, -1) = -4\sqrt{2}[1-1+\sqrt{2}] = -8 < 0 \Rightarrow P_3$ is a saddle point.

4) Minimize

$$f(x,y,z) = x^2 + y^2 + z^2$$

with the constraint that $x + y + z = S$.

$$F(x,y,z,\lambda) = x^2 + y^2 + z^2 - \lambda(x + y + z - S)$$

$F_x = 2x - \lambda \qquad F_x = 0 \Rightarrow x = \frac{\lambda}{2} \Rightarrow x = \frac{S}{3}$

$F_y = 2y - \lambda \qquad F_y = 0 \Rightarrow y = \frac{\lambda}{2} \Rightarrow y = \frac{S}{3}$

$F_z = 2z - \lambda \qquad F_z = 0 \Rightarrow z = \frac{\lambda}{2} \Rightarrow z = \frac{S}{3}$

$F_\lambda = -(x + y + z - S) \qquad F_\lambda = 0 \Rightarrow x + y + z = S$

$\Rightarrow \frac{3\lambda}{2} = S$

$\boxed{\lambda = \frac{2}{3}S}$

i.e. the minimum is

$$f\left(\frac{S}{3}, \frac{S}{3}, \frac{S}{3}\right) = \frac{S^2}{9} + \frac{S^2}{9} + \frac{S^2}{9} = \frac{S^2}{3}.$$

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5) $$\int_0^1 \int_{\frac{x}{2}}^{1-x/2} (2 - x - 2y)\, dy\, dx$$

$$= \int_0^1 (2y - xy - y^2) \Big|_{x/2}^{1-x/2} dx$$

$$= \int_0^1 \left[2\left(1-\frac{x}{2}\right) - x\left(1-\frac{x}{2}\right) - \left(1-\frac{x}{2}\right)^2\right] - \left[x - \frac{x^2}{2} - \frac{x^2}{4}\right] dx$$

$$= \int_0^1 1 - 2x + x^2 dx = \left(x - x^2 + \frac{x^3}{3}\right)\Big|_0^1 = 1 - 1 + \frac{1}{3} = \frac{1}{3}$$