2.4

28

$r \equiv$ flow rate
$C \equiv$ concentration
$V \equiv$ volume
$Q \equiv$ concentrate

$Q = C \cdot V$
$r = r_{in} = r_{out} = 5 \ \frac{gal}{min}$
$C_{in} = 0 \ \frac{lb}{gal}$
$V = 100$ gal

$$\frac{dQ}{dt} = r_{in} C_{in} - r_{out} C_{out} = r(C_{in} - C_{out}) = r\left(C_{in} - \frac{Q_{out}}{V}\right)$$

since the solution is well-stirred $Q_{out} = Q$

$$\frac{dQ}{dt} = -\frac{rQ}{V} = -\frac{5Q}{100} = -\frac{Q}{20}$$

(a) $\frac{dQ}{dt} + \frac{Q}{20} = 0$ • can use the integrating factor to solve

$p(t) = \frac{1}{20}$ $\quad u(t) = e^{\int \frac{1}{20} dt} = e^{\frac{1}{20}t}$
$q(t) = 0$

$$Q(t) = \frac{1}{e^{\frac{1}{20}t}} \int 0 \cdot e^{\frac{1}{20}t} dt = e^{-\frac{1}{20}t} \int 0 \cdot dt = e^{-\frac{1}{20}t}(0 + c) = Ce^{-\frac{1}{20}t}$$

@ $t = 0$ $\quad Q = 25$ lb.

$Q(0) = 25 = Ce^{-\frac{1}{20}(0)} = C \quad \Rightarrow \quad C = 25$

$\boxed{Q(t) = 25e^{-\frac{1}{20}t}}$

(b) let $Q(t) = 15$

$15 = 25e^{-\frac{1}{20}t} \Rightarrow \frac{15}{25} = e^{-\frac{1}{20}t} \Rightarrow \ln\left(\frac{3}{5}\right) = -\frac{1}{20}t \Rightarrow t = -20\ln\left(\frac{3}{5}\right)$

$\boxed{t \cong 10.22 \text{ minutes}}$

29) using notation from problem 28

$r = 5 \frac{gal}{min}$

$C_{in} = \frac{1}{2} \frac{lb}{gal}$

$V = 100$ gal ← 200 gallon tank is half full.

initial condition:

$Q(0) = 0$ lb.

$$\frac{dQ}{dt} = rC_{in} - r\frac{Q}{V} = \frac{5}{2} - \frac{5Q}{100} = \frac{5}{2} - \frac{Q}{20}$$

$$Q' + \frac{Q}{20} = \frac{5}{2}$$

$p(t) = \frac{1}{20}$ $\quad u(t) = e^{\int \frac{1}{20} dt} = e^{\frac{1}{20}t}$

$q(t) = 5/2$

$$Q(t) = \frac{1}{e^{\frac{1}{20}t}} \int \frac{5}{2} e^{\frac{1}{20}t} dt = \frac{5}{2} \frac{1}{e^{\frac{1}{20}t}} \int e^{\frac{1}{20}t} dt = \frac{5}{2} \frac{1}{e^{\frac{1}{20}t}} (20e^{\frac{1}{20}t} + C)$$

$$= \frac{100 e^{\frac{1}{20}t}}{2 e^{\frac{1}{20}t}} + \frac{5}{2} \frac{C}{e^{\frac{1}{20}t}} = 50 + Ce^{-\frac{1}{20}t}$$

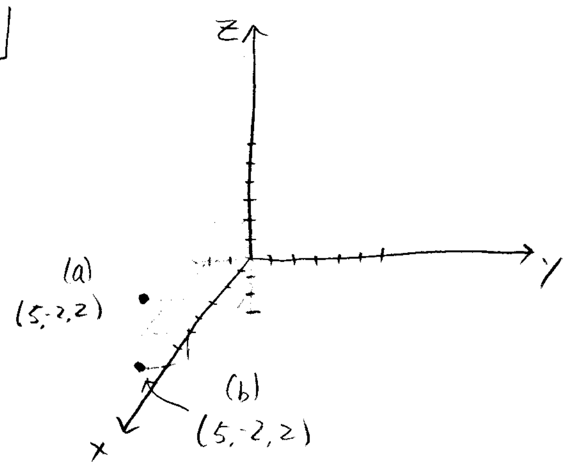
$$Q(0) = 0 = 50 + Ce^{-\frac{1}{20}(0)} = 50 + C \implies C = -50$$

$$\boxed{Q(t) = 50 - 50e^{-\frac{1}{20}t}}$$

$$Q(30) = 50 - 50e^{-3/2} = \boxed{38.84 \text{ lbs.}}$$

7.1

3)



6)

$$D = \sqrt{(-4-2)^2 + (-1+1)^2 + (1-5)^2}$$

$$= \sqrt{(-6)^2 + (0)^2 + (-4)^2}$$

$$= \boxed{\sqrt{52}}$$

10) the midpoint is basically the average between points

$$M = \left(\frac{4+8}{2}, \frac{0+8}{2}, \frac{20-6}{2}\right)$$

$$= \boxed{(6, 4, 7)}$$

18

$A = (5,3,4)$ $\quad d(AB) = \sqrt{(7-5)^2 + (1-3)^2 + (3-4)^2} = 3$

$B = (7,1,3)$ $\quad d(AC) = \sqrt{(3-5)^2 + (5-3)^2 + (3-4)^2} = 3$

$C = (3,5,3)$ $\quad d(BC) = \sqrt{(3-7)^2 + (5-1)^2 + (3-3)^2} = 4\sqrt{2}$

$\Rightarrow$ isosceles

19

$A = (-2,2,4)$ $\quad d(AB) = \sqrt{(-2+2)^2 + (2-2)^2 + (6-4)^2} = 2$

$B = (-2,2,6)$ $\quad d(AC) = \sqrt{(-2+2)^2 + (4-2)^2 + (8-4)^2} = 2\sqrt{5}$

$C = (-2,4,8)$ $\quad d(BC) = \sqrt{(-2+2)^2 + (4-2)^2 + (8-6)^2} = 2\sqrt{2}$

$\Rightarrow$ neither

26

$$\boxed{(x-4)^2 + (y+1)^2 + (z-1)^2 = 25}$$

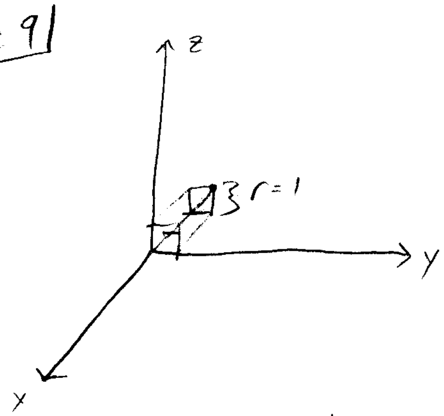
27 find the midpoint for the center

$$m = \left(\frac{2+0}{2}, \frac{0+6}{2}, \frac{0+0}{2}\right) = (1,3,0)$$

$$r = \sqrt{(2-1)^2 + (0-3)^2 + (0-0)^2} = \sqrt{10}$$

$$\boxed{(x-1)^2 + (y-3)^2 + z^2 = 10}$$

29



$$\boxed{(x+2)^2 + (y-1)^2 + (z-1)^2 = 1}$$

32 complete the square technique

$$x^2 + y^2 - 8y + z^2 = 0 \Rightarrow x^2 + (y-4)^2 - 16 + z^2 = 0$$

$$x^2 + (y-4)^2 + z^2 = 16$$

$$\Rightarrow \boxed{\begin{array}{l} r = 4 \\ C: (0,4,0) \end{array}}$$

33) $x^2 - 2x + 1 - 1 + y^2 + 6y + 9 - 9 + z^2 + 8z + 16 - 16 + 1 = 0$

$\Rightarrow (x-1)^2 + (y+3)^2 + (z+4)^2 = 25$

$$\boxed{\begin{array}{l} r = 5 \\ c: (1, -3, -4) \end{array}}$$

36) $4x^2 - 8x + 4y^2 + 16y + 4z^2 + 11 = 0$

$\Rightarrow x^2 - 2x + y^2 + 4y + z^2 + \frac{11}{4} = 0$

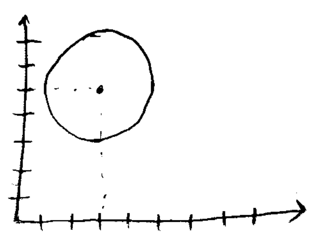
$\Rightarrow (x-1)^2 - 1 + (y+2)^2 - 4 + z^2 + \frac{11}{4} = 0$

$(x-1)^2 + (y+2)^2 + z^2 = \frac{9}{4}$

$$\boxed{\begin{array}{l} r = \frac{3}{2} \\ c: (1, -2, 0) \end{array}}$$

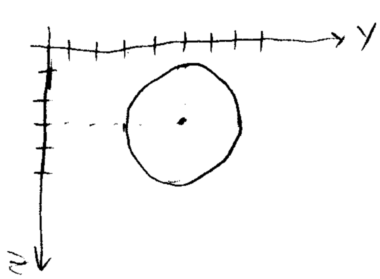
39) xy-trace $\Rightarrow z = 0$

$x^2 - 6x + y^2 - 10y + 30 = 0 \quad \Rightarrow (x-3)^2 + (y-5)^2 = 4$



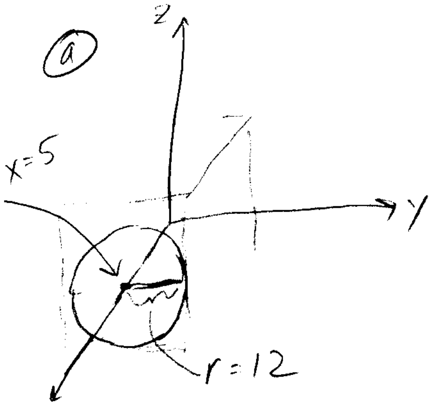
42) yz trace $\Rightarrow x = 0$

$y^2 - 10y + z^2 + 6z + 30 = 0 \quad \Rightarrow (y-5)^2 + (z+3)^2 = 4$

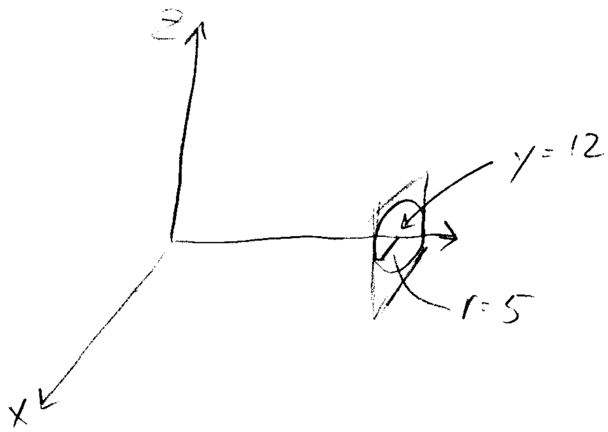


44 $y^2+z^2+25=169 \Rightarrow y^2+z^2=144$

(a)



(b) $x^2+z^2=25$

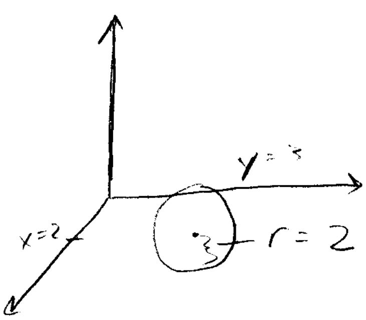


45

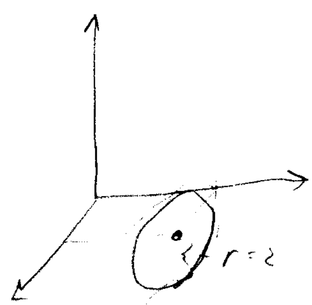
$x^2-4x+4-4+y^2-6y+9-9+z^2+9=0$

$(x-2)^2+(y-3)^2+z^2=4$

(a) $x=2 \Rightarrow (y-3)^2+z^2=4$



(b) $y=3 \Rightarrow (x-2)^2+z^2=4$



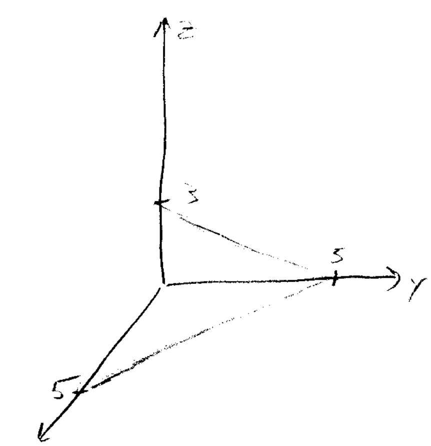
7.2

3 $3x+3y+5z=15$

$y=0, z=0 \Rightarrow x=5$

$x=0, z=0 \Rightarrow y=5$

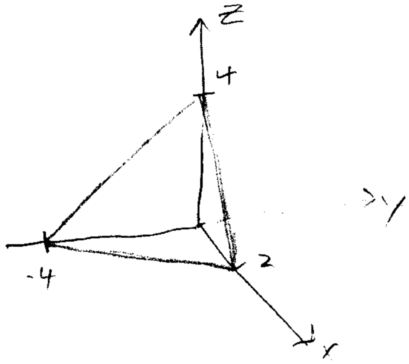
$x=0, y=0 \Rightarrow z=3$



6]

$2x - y + z = 4$

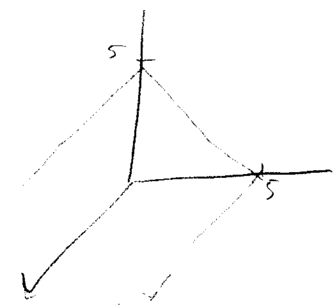
$y = 0, z = 0 \Rightarrow x = 2$
$x = 0, z = 0 \Rightarrow y = -4$
$x = 0, y = 0 \Rightarrow z = 4$



9]

$y + z = 5$

$y = 0, z = 0 \Rightarrow 0 = 5 \Rightarrow$ x doesn't have an intercept
$x = 0, z = 0 \Rightarrow y = 5$
$x = 0, y = 0 \Rightarrow z = 5$



14]

$3x + y - 4z = 3$
$-9x - 3y + 12z = 4$

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the planes are parallel

$\frac{3}{-9} = \frac{1}{-3} = \frac{-4}{12} \Rightarrow$ $\boxed{\text{parallel}}$

17]

$x + 2y = 3$
$4x + 8y = 5$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{1}{4} = \frac{2}{8} \Rightarrow$ $\boxed{\text{parallel}}$

20]

$2x - z = 1$
$4x + y + 8z = 10$

$(2)(4) + (0)(1) + (-1)(8) = 0 \Rightarrow$ $\boxed{\text{perpendicular}}$

22]

$3x + 3y + 2z - 6 = 0$
$(2, -1, 0)$

$$D = \frac{|(2)(3) + (-1)(3) + (0)(2) + (-6)|}{\sqrt{3^2 + 3^2 + 2^2}} = \boxed{\frac{3}{\sqrt{22}}}$$

29 $2x - 3y + 4z = 24$ $\quad D = \dfrac{|(2)(3) + (2)(-3) + (-1)(4) + (-24)|}{\sqrt{2^2 + (-3)^2 + 4^2}} = \boxed{\dfrac{28}{\sqrt{29}}}$

$(3, 2, -1)$

39 $x^2 - y - z^2 = 0$

xy-plane $\Rightarrow z = 0 \Rightarrow y = x^2$ parabola

$y = 1 \Rightarrow x^2 - z^2 = 1$ hyperbola

yz-plane $\Rightarrow x = 0 \Rightarrow y = -z^2$ parabola

42 $y^2 + z^2 - x^2 = 1$

xy-plane $\Rightarrow z = 0$ $\quad y^2 - x^2 = 1$ hyperbola

xz-plane $\Rightarrow y = 0$ $\quad z^2 - x^2 = 1$ hyperbola

yz-plane $\Rightarrow x = 0$ $\quad y^2 + z^2 = 1$ circle

45 $\dfrac{x^2}{(1/5)^2} + \dfrac{y^2}{(1/5)^2} - \dfrac{z^2}{(1)^2} = 5$

RHS > 0 and one term is negative

$\Rightarrow$ $\boxed{\text{hyperboloid of one sheet}}$

48 $z = \dfrac{x^2}{(1/2)^2} + \dfrac{y^2}{(1)^2}$

z is raised to the first power and the x and y terms are positive

$\Rightarrow$ $\boxed{\text{Elliptic Paraboloid}}$

50 $-\dfrac{x^2}{(1)^2} - \dfrac{y^2}{(2)^2} + \dfrac{z^2}{(1)^2} = 1$

RHS > 0 and two terms are negative

$\Rightarrow$ $\boxed{\text{hyperboloid of two sheets}}$

53 $\dfrac{x^2}{(1/3)^2} + \dfrac{y^2}{(1)^2} - \dfrac{z^2}{(1)^2} = 0$

RHS = 0 and one term is negative

$\Rightarrow$ $\boxed{\text{elliptic cone}}$

54 $y = \dfrac{x^2}{(2)^2} + \dfrac{z^2}{(2)^2}$

y is raised to the first power and the x and z terms are positive

$\Rightarrow$ $\boxed{\text{Elliptic Paraboloid}}$