7.3

2. $f(x,y) = 4 - x^2 - 4y^2$

a) $f(0,0) = 4 - 0^2 - 4(0)^2 = 4$ b) $f(0,1) = 4 - 0^2 - 4(1)^2 = 0$

c) $f(2,3) = 4 - 2^2 - 4(3)^2 = -36$ d) $f(1,y) = 4 - (1)^2 - 4y^2 = 3 - 4y^2$

e) $f(x,0) = 4 - x^2 - 4(0)^2 = 4 - x^2$ f) $f(t,1) = 4 - t^2 - 4(1)^2 = -t^2$

9. $A(P,r,t) = P\left[\left(1+\frac{r}{12}\right)^{12t} - 1\right]\left(1+\frac{12}{r}\right)$

a) $A(100, 0.10, 10) = 100\left[\left(1+\frac{0.10}{12}\right)^{12(10)} - 1\right]\left(1+\frac{12}{0.10}\right)$

b) $A(275, 0.0925, 40) = 275\left[\left(1+\frac{0.0925}{12}\right)^{12(40)} - 1\right]\left(1+\frac{12}{0.0925}\right)$

12. $g(x,y) = \int_x^y \frac{1}{t}\,dt$

a) $g(4,1) = \int_4^1 \frac{1}{t}\,dt = \ln|1| - \ln|4| = -\ln(4) = -2\ln(2)$

b) $g(6,3) = \int_6^3 \frac{1}{t}\,dt = \ln(3) - \ln(6) = \ln(3) - \ln(2\cdot 3)$

$= \ln(3) - (\ln(2) + \ln(3)) = -\ln(2)$

14. $f(x,y) = 3xy + y^2$

a) $f(x+\Delta x, y) = 3(x+\Delta x)y + y^2$

b) $$\frac{f(x, y+\Delta y) - f(x,y)}{\Delta y} = \frac{3x(y+\Delta y) + (y+\Delta y)^2 - (3xy + y^2)}{\Delta y}$$

$$= \frac{3x\Delta y + y^2 + 2y\Delta y + (\Delta y)^2 - y^2}{\Delta y} = \frac{3x\Delta y + 2y\Delta y + (\Delta y)^2}{\Delta y}$$

$$= 3x + 2y + \Delta y$$

15. $f(x,y) = \sqrt{16-x^2-y^2}$

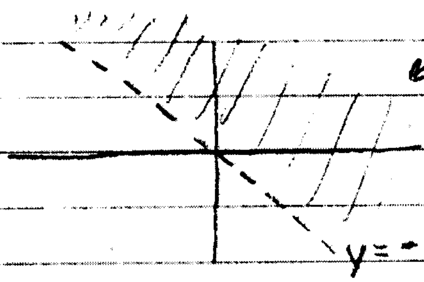
For Domain want $16-x^2-y^2 \geq 0 \Rightarrow 16 \geq x^2+y^2$.

Domain is the circle centered at the origin with radius 4

Range is all positive numbers less than or equal to 4.

18. $f(x,y) = \ln(x+y)$

For domain want $x+y>0 \Rightarrow x>-y$



Domain is shade region or when $x>-y$.

Range is all real numbers.

22. $f(x,y) = \frac{4y}{x-1}$

Domain is all points (x,y) except when $x=1$ Since $f(1,y)$ is undefined.

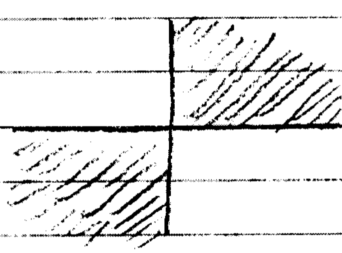
23. $f(x,y) = \frac{1}{xy}$

Want $xy \neq 0 \Rightarrow x \neq 0$ or $y \neq 0$

Domain is all points (x,y) such that $x \neq 0$ or $y \neq 0$

26. $f(x,y) = \sqrt{xy}$

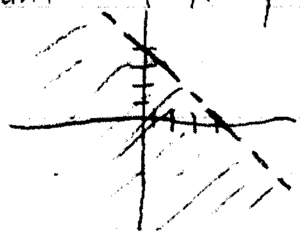
Want $xy \geq 0 \Rightarrow$ Both $x,y \geq 0$, Both $x,y \leq 0$.



Domain is the first and third quadrants, including the axises.

27. $g(x,y) = \ln(4-x-y)$

Want $4-x-y>0 \Rightarrow 4>x+y$



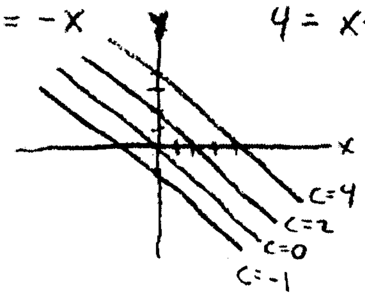
Domain is all (x,y) such that $4>x+y$.

30. $f(x,y) = e^{1-x^2+y^2}$
(d)

33. $z = x+y \qquad c = -1, 0, 2, 4$

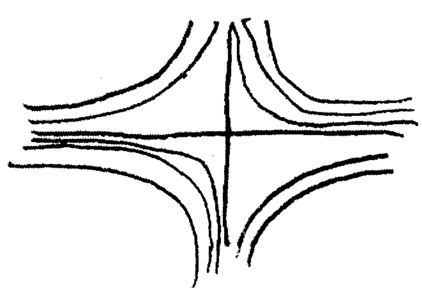
$-1 = x+y \Rightarrow y = -x-1 \qquad 2 = x+y \Rightarrow y = 2-x$

$0 = x+y \Rightarrow y = -x \qquad 4 = x+y \Rightarrow y = 4-x$



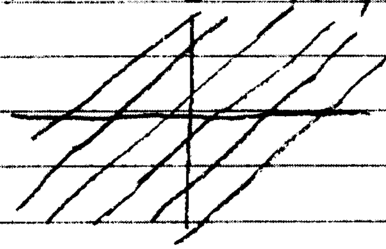
38. $z = e^{xy} \qquad c = 1, 2, 3, 4, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

$c = e^{xy} \Leftrightarrow \ln c = xy \Leftrightarrow \frac{\ln c}{x} = y$



40. $f(x,y) = \ln(x-y)$ $\qquad c = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$

$c = \ln(x-y) \iff e^c = x-y \iff y = x - e^c$



42. $f(x,y) = C\,x^a y^{1-a}$

x is labor

y is capital

$$f(2x,2y) = C(2x)^a(2y)^{1-a} = C\,2^a x^a 2^{1-a} y^{1-a}$$
$$= C\,2\,x^a y^{1-a}$$
$$= 2 f(x,y)$$

Section 7.4 — 23, 35, 36, 40, 41, 48, 54, 59, 62, 67

Key to HW

23. $f(x,y) = e^{3xy}$

$f_x(x,y) = e^{3xy}\cdot 3y \Rightarrow f_x(0,4) = e^0\cdot 3(4) = 12$

$f_y(x,y) = e^{3xy}\cdot 3x \Rightarrow f_y(0,4) = e^0\cdot 3(0) = 0$

35. $w = \ln(\sqrt{x^2+y^2+z^2}) = \frac{1}{2}\ln(x^2+y^2+z^2)$

$w_x = \frac{1}{2}\cdot\frac{1}{x^2+y^2+z^2}\cdot 2x = \frac{x}{x^2+y^2+z^2}$

$w_y = \frac{1}{2}\frac{1}{x^2+y^2+z^2}\cdot 2y = \frac{y}{x^2+y^2+z^2}$

$w_z = \frac{1}{2}\frac{1}{x^2+y^2+z^2}\cdot 2z = z/x^2+y^2+z^2$

$w_x(3,0,4) = 3/9+16 = 3/25$

$w_y(3,0,4) = 0$, $w_z(3,0,4) = \frac{4}{25}$.

36. $w = (1-x^2-y^2-z^2)^{-1/2}$

$w_x = -\frac{1}{2}(1-x^2-y^2-z^2)^{-3/2}(-2x)$

$w_y = -\frac{1}{2}(1-x^2-y^2-z^2)^{-3/2}(-2y)$

$w_z = -\frac{1}{2}(1-x^2-y^2-z^2)^{-3/2}(-2z)$

at $P=(0,0,0)$ each of these is zero.

$w_x(0,0,0) = w_y(0,0,0) = w_z(0,0,0) = 0.$

[redacted] [redacted]

40 $f(x,y) = 3x^3 - 12xy + y^3$

$f_x(x,y) = 9x^2 - 12y$. So $f_x = 0 \Rightarrow 12y = 9x^2$

$y = \frac{3}{4}x^2$

$f_y(x,y) = -12x + 3y^2$

So $f_y = 0 \Rightarrow 12x = 3y^2$

$x = \frac{1}{4}y^2$

Together we have

$$y = \frac{3}{4}x^2 = \frac{3}{4}\left(\frac{1}{4}y^2\right)^2$$

$$\Rightarrow \frac{3}{64}y^4 - y = 0$$

$$y\left(1 - \frac{3}{64}y^3\right) = 0$$

$$\Rightarrow y = 0 \quad \text{or} \quad 1 - \frac{3}{64}y^3 = 0$$

$$\frac{3}{64}y^3 = 1$$

$$y^3 = \frac{64}{3}$$

$$y = \frac{4}{\sqrt[3]{3}}$$

Points

$P_1 = (0,0)$ and $P_2 = \left(\frac{1}{4}\left(\frac{4}{\sqrt[3]{3}}\right)^2, \frac{4}{\sqrt[3]{3}}\right)$

41. $f(x,y) = \frac{1}{x} + \frac{1}{y} + xy$

$f_x(x,y) = \frac{-1}{x^2} + y$ $\quad f_x = 0 \Rightarrow y = \frac{1}{x^2}$

$f_y(x,y) = \frac{-1}{y^2} + x$ $\quad f_y = 0 \Rightarrow x = 1/y^2$

Together

$$y = \frac{1}{x^2} = \frac{1}{\left(\frac{1}{y^2}\right)^2} = y^4$$

$$\Rightarrow \quad y^4 - y = 0 \quad \Rightarrow \quad y = 0 \text{ or } y = 1$$

$y = 0$ is not in the domain.
Thus $y = 1$ is the only reasonable solution.
Solving for x $\Rightarrow$ $P_1 = (1,1)$.

48. $z = \frac{x}{y}$, $z_x = \frac{1}{y}$, $z_y = \frac{-x}{y^2}$

At the point $(3,1,3)$, $z_x = 1$ and $z_y = -3$

54. $z = \frac{x^2 - y^2}{2xy}$

Calculate

$4x^2y - 2x^2y + 2y^3$

$$z_x = \frac{2xy(2x) - (x^2-y^2)(2y)}{4x^2y^2} = \frac{2y^3 + 2x^2y}{4x^2y^2} = \frac{\cancel{2y}(y^2+x^2)}{\cancel{2y}\cdot 2x^2y}$$

$$z_y = \frac{2xy(-2y) - (x^2-y^2)(2x)}{4x^2y^2} = \frac{-2x^3 - 2xy^2}{4x^2y^2} = \frac{-\cancel{2x}(x^2+y^2)}{\cancel{2x}\cdot 2xy^2}$$

$$z_{xy} = \frac{\partial}{\partial y}\left(\frac{x^2+y^2}{2x^2y}\right) = \frac{2x^2y(2y) - (x^2+y^2)(2x^2)}{4x^4y^2} = \frac{2x^2y^2 - 2x^4}{4x^4y^2}$$

$$z_{yx} = \frac{\partial}{\partial x}\left(\frac{-(x^2+y^2)}{2xy^2}\right) = -\frac{2xy^2(2x) - (x^2+y^2)(2y^2)}{4x^2y^4} = \frac{-2x^2y^2 + 2y^4}{4x^2y^4}$$

Clearly

$$z_{xy} = \frac{2x^2(y^2-x^2)}{2x^2 \cdot 2x^2y^2} = \frac{y^2-x^2}{2x^2y^2}$$

$$z_{yx} = \frac{2y^2(y^2-x^2)}{2y^2 \cdot 2x^2y^2} = \frac{y^2-x^2}{2x^2y^2}$$

These are equal

59. $z = \frac{xy}{x-y}$

$$z_x = \frac{(x-y)y - xy(1)}{(x-y)^2} = \frac{-y^2}{(x-y)^2}$$

$$z_y = \frac{(x-y)x - xy(-1)}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$z_{xx} = \frac{2y^2}{(x-y)^3}, \quad z_{xy} = \frac{(x-y)^2(-2y) - (-y^2)2(x-y)(-1)}{(x-y)^4}$$

$$z_{yy} = \frac{2x^2}{(x-y)^3}, \quad z_{yx} = \frac{(x-y)^2(2x) - x^2\,2(x-y)}{(x-y)^4}$$

62. $z = xe^y + ye^x$

$z_x = e^y + ye^x$, $z_y = xe^y + e^x$

$z_{xx} = ye^x$, $z_{yy} = xe^y$

$z_{xy} = e^y + e^x$, $z_{yx} = e^y + e^x$

5

67. $C = 10\sqrt{xy} + 149x + 189y + 675$

$$\frac{\partial C}{\partial x} = 10 \cdot \frac{1}{2}(xy)^{-1/2}\, y + 149$$

$$\frac{\partial C}{\partial y} = 10 \cdot \frac{1}{2}(xy)^{-1/2}\, x + 189$$

$$\frac{\partial C}{\partial x}(120,160) = 5 \cdot \frac{160}{\sqrt{120 \cdot 160}} + 149$$

$$\frac{\partial C}{\partial y}(120,160) = 5 \cdot \frac{120}{\sqrt{120 \cdot 160}} + 189.$$