# 119B: Ordinary Differential Equations

**Department of Mathematics, UC Davis, Spring 2012**

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# Course Syllabus

The course will focus on nonlinear phenomena in the theory of ODEs and dynamical systems. The first part of the course is devoted to *Hamiltonian mechanics*, a deep and beautiful way of looking at the equations of motion of mechanical systems that has also heavily influenced other fields of math, notably geometry and optimization problems, as well as theoretical physics.

The second part of the course will focus on *chaotic systems*, both in continuous and discrete time. We will examine some of the weird phenomena associated with chaos and consider ways in which the complexity can be dealt with by focusing on statistical aspects of the system instead of exact prediction—this leads naturally to a branch of mathematics known as *ergodic theory*.

The third part of the course is devoted to *control theory*, which studies how a dynamical system can be stabilized around an ordinarily unstable region of the phase space by applying an external controlling influence. A main idea is to use *feedback* (we all do this, e.g., when riding a bicycle!). This is an extremely applied part of ODE theory, with thousands of applications in science, engineering and industry (aerospace, automotive, chemical process control, nuclear plants, medical etc.). It is also a lot of fun, since we get to tweak the ODE rather than just analyzing a static equation that is handed to us by someone else.

### Detailed list of topics

- **Hamiltonian mechanics**

  1. Hamilton's equations
  2. The Euler-Lagrange equation and the Lagrangian formalism
  3. Equivalence of the Langrange and Hamilton formalisms
  4. Connection to Newton's equations of motion; examples
  5. Conservation of area: Liouville's theorem
  6. The principle of least action

7. Introduction to the calculus of variations; geodesics, the brachistochrone problem
8. The one-body problem in a central gravitational field
9. The two-body problem
10. Analytic solution of the simple pendulum using elliptic integrals; application to computing the speed of a wave of falling dominoes.

- **Chaos, discrete-time dynamics and ergodic theory**

  1. The logistic map: bifurcations, emergence of chaos
  2. Interval maps: doubling map, tent map, circle rotations and more
  3. Statistical understanding of chaos: invariant measures and ergodicity
  4. Introduction to ergodic theory: the ergodic theorem and its applications
  5. More examples: Billiard systems, Hamiltonian flows, etc.
  6. The Lorenz equations, strange attractors

- **Control theory**

  1. Basic principles of control; examples
  2. Control without feedback: the inverted pendulum with vertically oscillating base; alternating gradient focusing in particle accelerators
  3. Simple control with feedback: electromagnetic levitation by binary switching
  4. Optimal control with feedback: the inverted pendulum with moving base (a.k.a. Segway, self-stabilizing robot); the Linear Quadratic Regulator, solution using the Ricatti equation.

- **Optional topics, if time permits**

  1. Fractals
  2. Fractal dimension