

Math 25 — Homework Assignment #1

Homework due: Tuesday 4/5/11 at beginning of discussion section

Homework guidelines. In answering the questions, you are asked to communicate your knowledge and understanding to your reader. Therefore all answers (even to yes/no questions) must be explained in clear language using full sentences. Think of the communication aspect as part of the assignment; it is not enough that you “figured it out” in your head, your answer also has to convince the reader of this to get full credit.

Reading material. Before beginning work on the problems, read sections A.4, A.5, A.6, A.7 and A.9 in the textbook.

Problems

1. Prove that if a natural number n is divisible by 3 and by 5 then it is divisible by 15.
2. Prove or disprove¹ the following claim: any natural number n that is divisible by 6 and by 8 is divisible by 48. Which proof technique did you use?
3. Let p be a prime number (a positive integer which is divisible only by 1 and itself). Prove that \sqrt{p} is irrational.²
4. Prove the following assertion by contraposition: if x is irrational, then $x + r$ is irrational for all rational numbers r . (For guidance, see note 391 on page A-19 at the end of Appendix A in the textbook).
5. Every prime number greater than 2 is odd. Is the converse true?
6. Prove that there are infinitely many prime numbers. (Use proof by contradiction; for guidance, see note 390 on page A-19 at the end of Appendix A in the textbook).

¹“disprove” means to prove the negation of the claim, which in this case is the assertion that not all natural numbers that are divisible by 6 and 8 are divisible by 48.

²You may use the following known fact about prime numbers: if p is a prime number and a, b are integers such that $a \cdot b$ is divisible by p , then either a is divisible by p or b is divisible by p .