Math 25 — Homework Assignment #2

Homework due: Tuesday 4/12/11 at beginning of discussion section

Homework guidelines. [1] As you may have discovered in last week's assignment, writing a good proof is not easy. For this week's assignment I suggest the following steps: 1. Spend a few minutes contemplating how to present your argument. 2. Write a draft of the proof on a separate sheet of paper. 3. Go and do something else for 5 minutes. 4. Read over your proof and consider if you would be satisfied if your instructor were to write such a proof on the blackboard. If not, go back to step 1. If yes, copy your proof into the pages you are planning to submit and proceed to the next problem.

Reading material. Read sections A.8 and 2.3 in the textbook.

Problems

1. Prove by induction that for every $n = 1, 2, 3, \ldots$
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
(for a hint, see note 392 at the end of Appendix A in the textbook).

2. Let x be a positive real number. Prove by induction that for every $n \in \mathbb{N}$,
$$(1+x)^n \geq 1 + nx.$$

3. Prove by induction that for every $n \in \mathbb{N}$ the number $7^n - 4^n$ is divisible by 3.

4. Find and explain the flaw in the proof by induction that was presented in class of the claim that all giraffes are the same height (see also problem A.8.9 in the textbook).

5. Answer problems A.9.1 and A.9.2 in the textbook.

6. Prove that the set $\mathbb{N} \times \mathbb{N} = \{\text{all pairs } (m, n) \text{ where } m, n \in \mathbb{N}\}$ is countable. That is, show that one can write down a list of pairs $(m_1, n_1), (m_2, n_2), (m_3, n_3), \ldots$ covering all the elements of $\mathbb{N} \times \mathbb{N}$ (the list may contain repetitions but any pair (m, n) of natural numbers must appear at least once).[2]

[1] Last week's HW guidelines also still apply — refresh your memory by rereading them.

[2] You do not need to write a precise formula for the general term (m_k, n_k) of this list, as long as you provide sufficient detail about how to construct the list that convinces the reader that you understand what you are talking about.