Math 25 — Homework Assignment #6

Homework due: Tuesday 5/10/11 at beginning of discussion section

Reading material. Read section 2.8 in the textbook.

Problems

1. Let $(s_n)_{n=1}^\infty$ and $(t_n)_{n=1}^\infty$ be sequences that converge to limits $S = \lim_{n\to\infty} s_n$ and $T = \lim_{n\to\infty} t_n$. We proved in class a theorem that says that the sequence $s_n + t_n$ converges to the limit $S + T$. By imitating the proof of that result, show that the sequence $s_n - t_n$ converges to $S - T$.

2. Let $(s_n)_{n=1}^\infty$ and $(t_n)_{n=1}^\infty$ be defined by
$$s_n = \sqrt{n+1}, \qquad t_n = \sqrt{n}.$$
Explain why the result in problem 1 above does not apply. Prove however that
$$\lim_{n\to\infty} (s_n - t_n) = 0.$$
Hint: Use the identity $\sqrt{a} - \sqrt{b} = \frac{a-b}{\sqrt{a}+\sqrt{b}}$ and the squeeze theorem.

3. Compute the following limits. For each computation, provide a brief explanation how you obtained your answer from known results.
   (a) $\lim_{n\to\infty} \frac{1}{n^3}$
   (b) $\lim_{n\to\infty} \frac{3n^2-1}{n+1}$
   (c) $\lim_{n\to\infty} \frac{3n^2+\sin(n)}{n^2}$
   (d) $\lim_{n\to\infty} \frac{5n^4-2n^2+n+1}{n^2(n^2+1)}$

4. Let $(s_n)_{n=1}^\infty$ and $(t_n)_{n=1}^\infty$ be sequences. Decide which of the following statements are true. In each case, provide a proof or a counterexample.
   (a) If $(s_n)_{n=1}^\infty$ and $(t_n)_{n=1}^\infty$ are both divergent then so is $(s_n + t_n)_{n=1}^\infty$.
   (b) If $(s_n)_{n=1}^\infty$ and $(t_n)_{n=1}^\infty$ are both divergent then so is $(s_n \cdot t_n)_{n=1}^\infty$.
   (c) If $(s_n)_{n=1}^\infty$ and $(s_n + t_n)_{n=1}^\infty$ are both divergent then so is $(t_n)_{n=1}^\infty$.
   (d) If $(s_n)_{n=1}^\infty$ is convergent then so is $(1/s_n)_{n=1}^\infty$.