

MAT 280: Selected topics in probability and combinatorics

Course Syllabus

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Course description

The course will cover selected parts of my book, “*The Surprising Mathematics of Longest Increasing Subsequences*”¹, and focuses on mathematics related to the problem of longest increasing subsequences. The basic question is the following:

In a randomly ordered list of n numbers, what is the maximal length of an *increasing subsequence* that we can extract?

For example, if the list is $(8, 1, 3, 2, 6, 4, 7, 5)$, the answer is 4, since $(1, 2, 4, 5)$ is an increasing subsequence of length 4, and there are no increasing subsequences of length 5. It turns out that trying to find a precise answer to this question as $n \rightarrow \infty$ leads to fascinating and deep mathematics which relates to many other interesting ideas and questions in probability, combinatorics, random matrix theory, statistical physics and other areas.

The likely topics I will cover are taken from chapters 1, 2 and 4 in the book:

1. Combinatorics of Young diagrams and Young tableaux. The Robinson-Schensted algorithm.
2. Solution of the problem of the typical length of longest increasing subsequences in a random permutation.
3. Fluctuations of the length of longest increasing subsequences in a random permutation: the Tracy-Widom distribution and Baik-Deift-Johansson theorem.
4. The corner growth process: randomly growing Young diagrams and their limit shapes. Connection to hydrodynamic limits in the exclusion process.

Prerequisites. MAT/STA235A (concurrently).

¹The book can be downloaded from <http://www.math.ucdavis.edu/~romik/lisbook>.