Homework due. Tuesday 11/01/11 at discussion section.

Reading material. Read Sections 6.5–6.6 in the textbook.

Problems

1. Compute the coordinate vector $[v]_B$, where:
   
   (a) $v = (1, 0, 1)$, $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$.
   
   (b) $v = (1, 0, 1)$, $B = \{(1, 0, 1), (1, 0, 0), (0, 1, 0)\}$.
   
   (c) $v = z^3 - 2z$, $B = \{z + 1, z - 1, z^2, z^3\}$ in the space $P_3$ of polynomials of degree $\leq 3$.

2. Compute the representation matrix $M(T)^B_C$, where:
   
   (a) $T(x, y) = (x + 10y, -x)$, $B = C = \{(1, 0), (0, 1)\}$.
   
   (b) $T(x, y, z) = (z, y, 3x)$, $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$, $C = \{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$
   
   (c) $T(x, y, z) = (z, y, 3x)$, $B = \{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$, $C = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

3. Solve calculational exercises 1(d), 1(e), 2(c), 3, 5 in Chapter 6.

4. Let $U, V, W$ be finite-dimensional vector spaces, and let $S : U \to V$, $T : V \to W$ be linear transformations. The goal of this problem is to prove the inequality:

   $$\dim(\ker(T \circ S)) \leq \dim(\ker(S)) + \dim(\ker(T)),$$

   where $T \circ S : U \to W$ denotes the composition of the two transformations. Prove this by using the following steps:

   (a) Denote $H = \ker(T \circ S)$ (a linear subspace of $U$), and define a linear transformation $R : H \to V$ by $R(v) = S(v)$ (i.e., it is the same as $S$, but its domain is a subspace of the domain of $S$; sometimes $R$ defined in this way will be referred to as the restriction of $S$ to $H$). Show that $\ker(S) \subseteq H$, and explain why this implies that $\ker(R) = \ker(S)$.

   (b) Show that $\text{range}(R) \subseteq \ker(T)$.

   (c) Apply the dimension formula (Theorem 6.5.1 in the textbook) for a suitable linear transformation to deduce the inequality stated at the beginning of the question.