Homework due.  Tuesday 11/15/11 at discussion section.

Reading material.  Read Chapter 8 in the textbook.

Problems

1.  (a) Compute the composition $\sigma \circ \pi$ of permutations, where:
\[ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 6 & 2 & 4 & 5 \end{pmatrix}, \quad \pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 1 & 2 & 3 & 4 \end{pmatrix} \]
\[ \text{ii. } \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \quad \pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \]

(b) Find the inverses of the permutations $\sigma$ and $\pi$ in part (a)-i. above.

2.  For each of the following matrices, compute its determinant and its adjoint matrix.  If the matrix is invertible, use the adjoint matrix to find its inverse.  (As usual, it is strongly recommended to check your answer by multiplying the matrix by the inverse matrix you found.)

(a) \[ \begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 1 \\ 0 & -1 & 3 \end{pmatrix} \]

(b) \[ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & -5 \\ 0 & 0 & -1 & 3 \end{pmatrix} \]

3.  Let $A$ be a square matrix of order 4.  We perform the following sequence of elementary row operations on $A$:
1. Subtract 3 times row 1 from row 2.
2. Multiply row 3 by 1/5.
3. Swap rows 1 and 4.
4. Add 10 times row 1 to row 2.
5. Swap rows 1 and 2.

After performing these operations, we get the new matrix $B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$.

(a) Find a matrix $D$ such that $B = DA$.
\[ \text{Hint: Each of the elementary row operations can be represented as the operation of multiplying the current matrix from the left by an “elementary matrix”}. \]

(b) Find $\det(A)$.  

4. Prove that for any \( n \), the number of permutations of order \( n \) with sign 1 is equal to the number of permutations with sign \(-1\).

**Hint:** find a way of matching up the permutations of positive and negative signs in pairs, i.e., for each permutation with positive sign find a way to associate with it a permutation with negative sign, such that each “negative” permutation is associated with exactly one “positive” permutation.

5. Solve the following exercises from the textbook:
   
   (a) Calculational exercises 1 and 6 in Chapter 8.
   (b) Proof-writing exercises 2 and 3 in Chapter 8.