General information about Midterm Exam 2

Date of the exam. The exam will be held on Wednesday, Feb. 24 at the usual time and place of the class lecture.

Material to be covered. All of Chapter 6, except section 6.7 and mass, moments and center of mass computations for two-dimensional objects. (Mass and center of mass computations for one-dimensional objects, i.e., thin wires and rods, *are* covered.)

Tips for studying. As with the first midterm exam, the questions will test both your ability to obtain a correct final answer, and your ability to effectively communicate how you derived the final answer (the "work" part). Grading will take both of these elements into account. That means the solution sheets are expected to be readable accounts of your solution; use verbal explanations where appropriate, and clearly demarcate solutions to different questions. It also means that you should take care to use correct notation and not just use some arbitrary symbols or nonstandard notation that you were using privately when solving problems on your own. Remember, math is a language, and part of the goal of the course is to teach you that language so that you can efficiently communicate mathematical ideas to and from other people. For example:

"$\int f(x)$" is incorrect notation, but "$\int f(x)\,dx$" is correct;

"$\int_0^\pi \cos(x)\,dx = \int \sin(x)\Big|_0^\pi$" is incorrect, whereas "$\int_0^\pi \cos(x)\,dx = \sin(x)\Big|_0^\pi$" is correct.

Practice questions. The practice questions below, as well as the homework problems, are representative of the kinds of questions you can expect to appear in the midterm exam, although (in contrast to exam questions) they may require a calculator and may take longer to solve than exam questions. Also keep in mind they are simply sample questions and are not meant to be indicative of exactly which topics will be emphasized in the exam. You are advised to master *all* the topics.

Practice questions for Midterm Exam 2

1. Let C denote the curve $y = 2 - |x|$, $-1 \le x \le 1$.

(a) Compute the volume of the solid of revolution formed by revolving the region bounded between C and the x-axis about the x-axis.

(b) Compute the surface area of the surface of revolution of C about the x-axis.

(c) Compute the arc length of C.

2. The parametric curve

$$x = \frac{2}{3}t^{3/2}, \quad y = 2\sqrt{t}, \qquad 0 \le t \le \sqrt{3}$$

is revolved about the y-axis. Find the area of the resulting surface.

3. A thin rod of length 2 meters is made of a material whose composition changes along the length of the rod. The density at distance x from the left end of the rod is $f(x) = 100 + x^2/200$ grams per centimeter. Find the total mass of the rod and its center of mass.

4. (a) A spring satisfies Hooke's law with an elastic constant $k_1 = 20$ lb/ft. Compute the work needed to stretch the spring from its original length of 3 feet to 4 feet. Specify the units for the answer.

(b) Compute the answer to the same question, under the additional assumption that after stretching the spring by 0.5 feet, the spring suffers a partial breakdown of its elastic material, causing the elastic constant to drop in value to $k_2 = 8$ lb/ft. In other words, the spring's resistive force $F(x)$ as a function of the amount x by which it was stretched is given by

$$F(x) = \begin{cases} k_1 x & \text{if } x < 0.5, \\ k_2 x & \text{if } x \ge 0.5. \end{cases}$$

5. A water tank is shaped like the surface of revolution about the y-axis of the curve $y = 3x^2$, $0 \leq x \leq 2$, (both x and y have meter units). It is filled with water, whose density is 1000 kg/m^3, up to a height of 8 meters.

(a) Compute the total mass of the water in the tank. (Hint: there is a simple relation between the mass and the volume...)

(b) Compute the amount of work that needs to be expended to pump all the water out over the sides of the tank. Note that you need to know the height of the tank, and use the fact that the Earth's gravity pulls down a mass of 1 kilogram with a force equal (roughly) to 9.8 Newtons – this value is the Earth's gravitational constant, commonly denoted g.

(∗) **6.** The famous Gateway Arch in Saint Louis, Missouri is described mathematically (ignoring its third dimension) as the two-dimensional region located above the x-axis and bounded between the two curves

$$\begin{aligned} y_{\text{outer}} &= H - \frac{a}{2}\left(e^{x/a} + e^{-(x/a)} - 2\right), \\ y_{\text{inner}} &= h - \frac{c}{2}\left(e^{x/c} + e^{-(x/c)} - 2\right). \end{aligned}$$

See Figure 1 below. When x and y are measured in feet, the parameters H, h, a, c are given by

$$\begin{aligned} H &= 630 \text{ ft}, \quad \text{(the height of the arch)}, \\ h &= 615 \text{ ft}, \\ a &= 127.7 \text{ ft}, \\ c &= 101.4 \text{ ft}. \end{aligned}$$

It can be checked that y_{outer} is positive exactly when $-L < x < L$, where $L \approx 315$ ft (in other words the span of the arch is $2L \approx 630$ ft), and similarly y_{inner} is positive when $-K < x < K$, where $K \approx 263$ ft (i.e., the "inner span" is 526 feet).

(a) Find the arc length of the outer curve.

(b) Find the area of the arc (again treating it as a two-dimensional object). Note that in the ranges where $K \le |x| \le L$ the inner curve lies below the x-axis, so those ranges need to be treated separately from the range $-K \le x \le K$.

Note. Clearly, this question requires the use of a calculator...

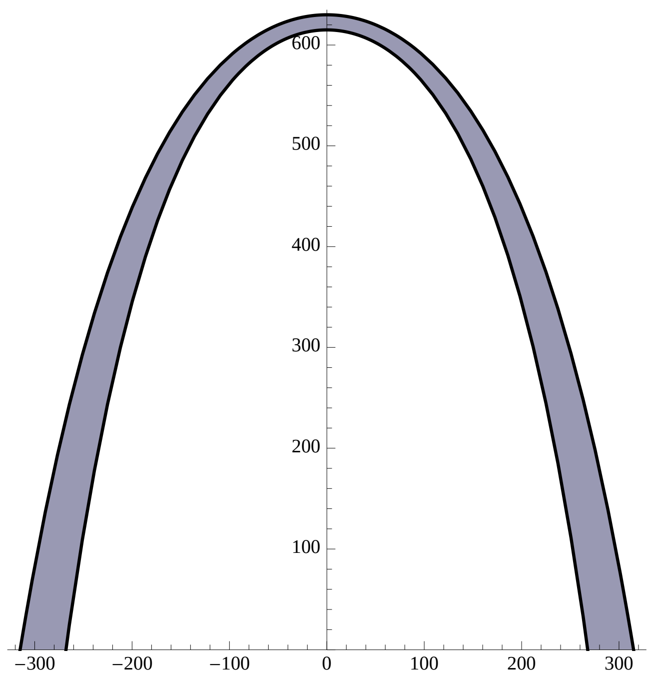

Figure 1: The mathematical shape of the Gateway Arch